Grade 12 Calculus

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THE CHANGE IN EDUCATION

Real World

- Science and Medicine
- Technology and Engineering
- Business and Economics
- History and Philosophy
- Arts and Athletics

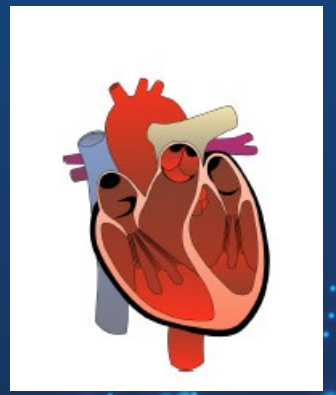
Real World

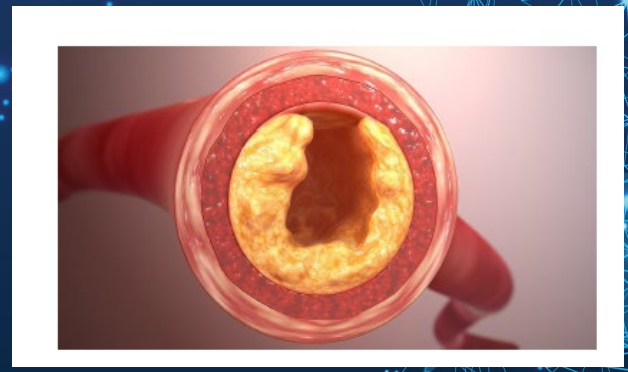
- Science and Medicine

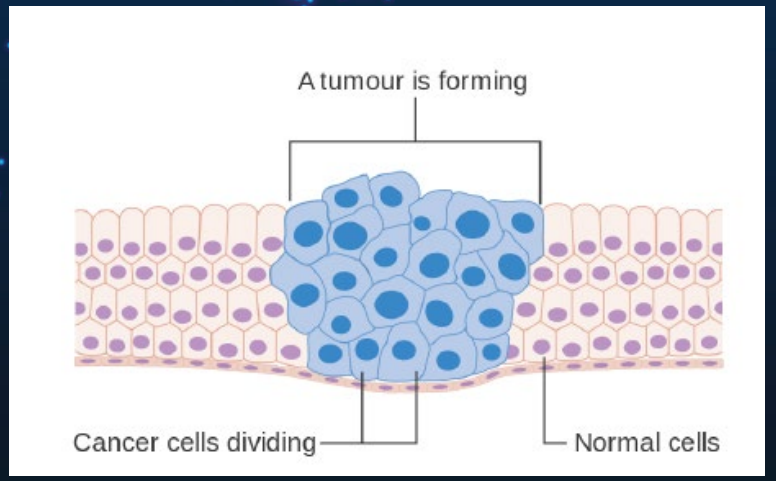
Cardiac Output

Understanding Plaque Buildup

Modeling Tumor Growth







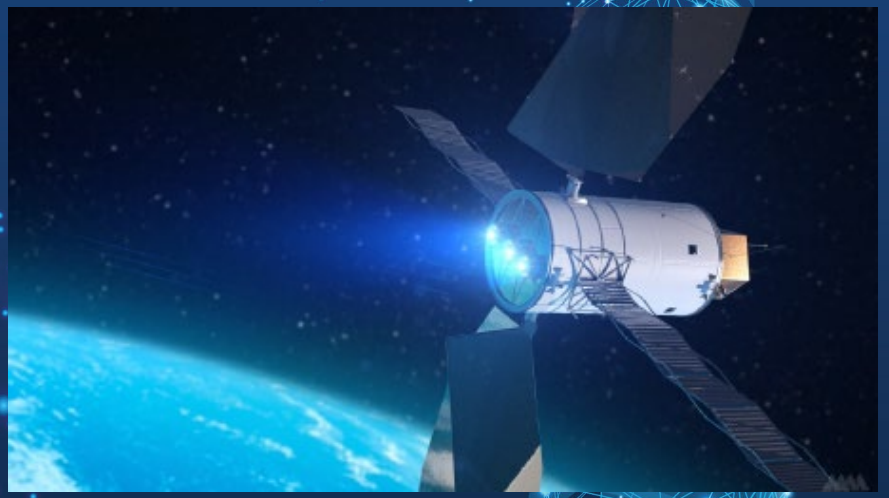
Real World

- Technology and Engineering

Rocket Propulsion

3D Computer Graphics

Physics Engines in Video Games



Real World

- Business and Economics

Maximizing Profit

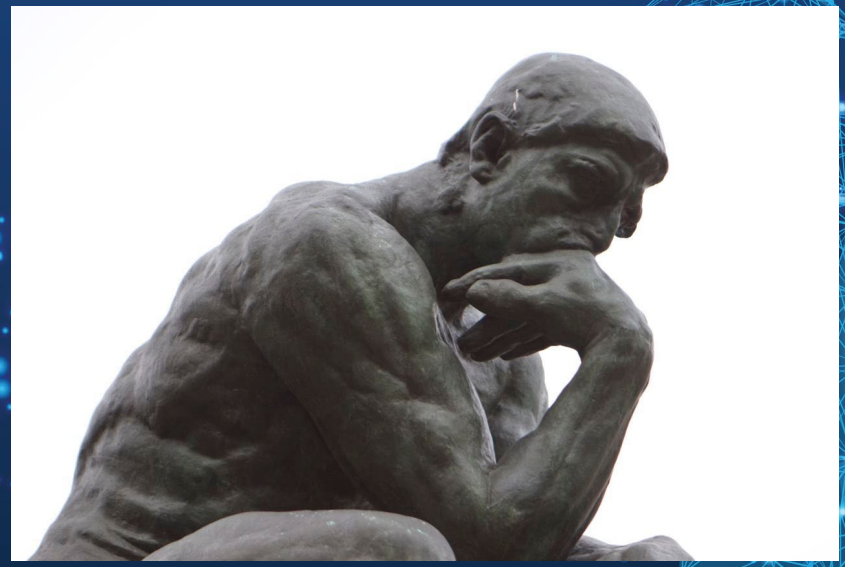
Continuously Compounded Interest

$$\lim_{n\to\infty} P\left(1+\frac{r}{n}\right)^{nt} = P\left[\lim_{n\to\infty}\left(1+\frac{r}{n}\right)^{n}\right]^{t}$$
$$= P\left[e^{r}\right]^{t}$$
$$= Pe^{rt}$$

Real World

- History and Philosophy

A Failure of Intuition

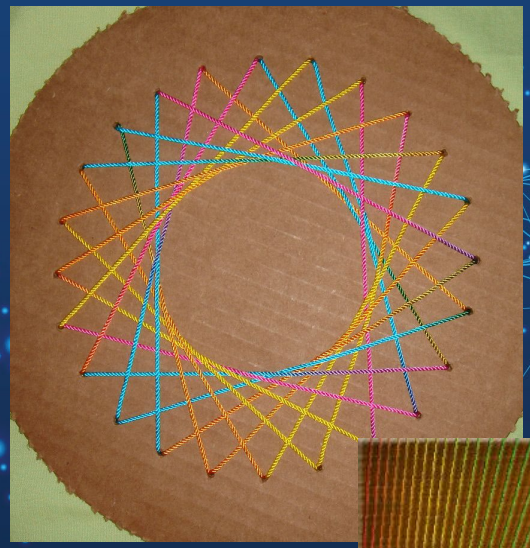


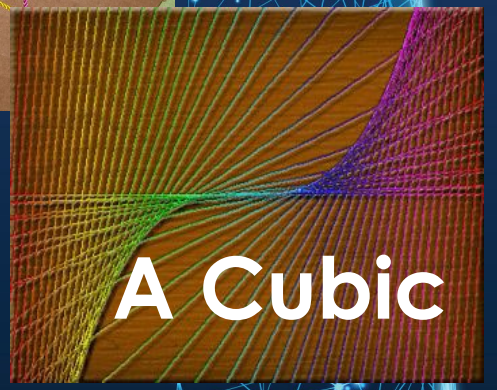
Real World

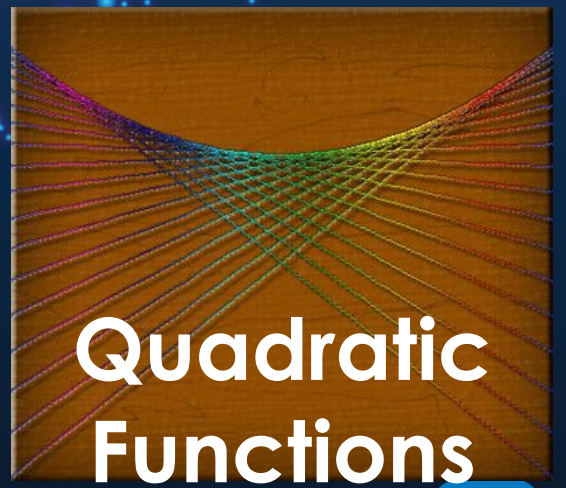
- Arts and Athletics

Derivatives in String Art

Calculating Horsepower of an Offensive Lineman







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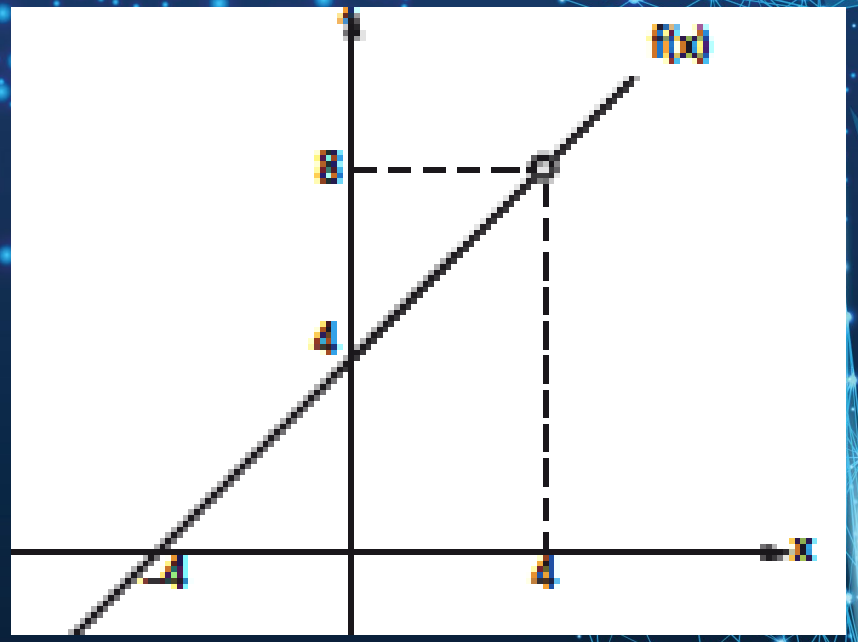
Limits

Straight lines

$$f(x) = \frac{x^2 - 16}{x - 4}$$

Limit $x = 4$ (boulder)

$$f(x) = \lim_{x \to 4} \frac{x^2 - 16}{x - 4}$$



Limits

Level 1 Questions

$$f(x) = \lim_{x \to -3} 3x$$

Level 2 Questions

$$f(x) = \lim_{x \to -2} \frac{x^2 - 4}{3x}$$

Level 3 Questions

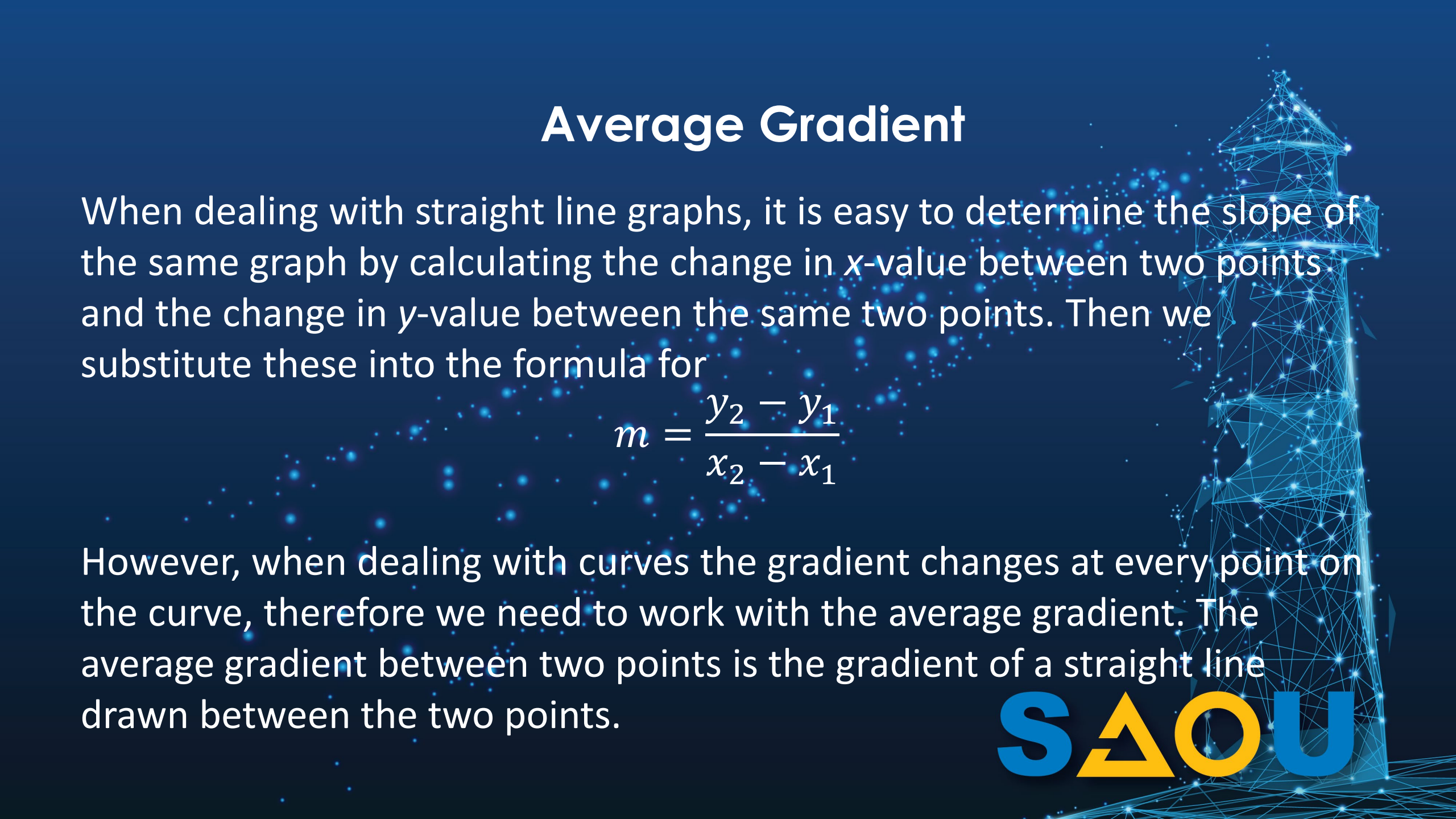
$$f(x) = \lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

Average Gradient

When dealing with straight line graphs, it is easy to determine the slope of the same graph by calculating the change in *x*-value between two points and the change in *y*-value between the same two points. Then we substitute these into the formula for

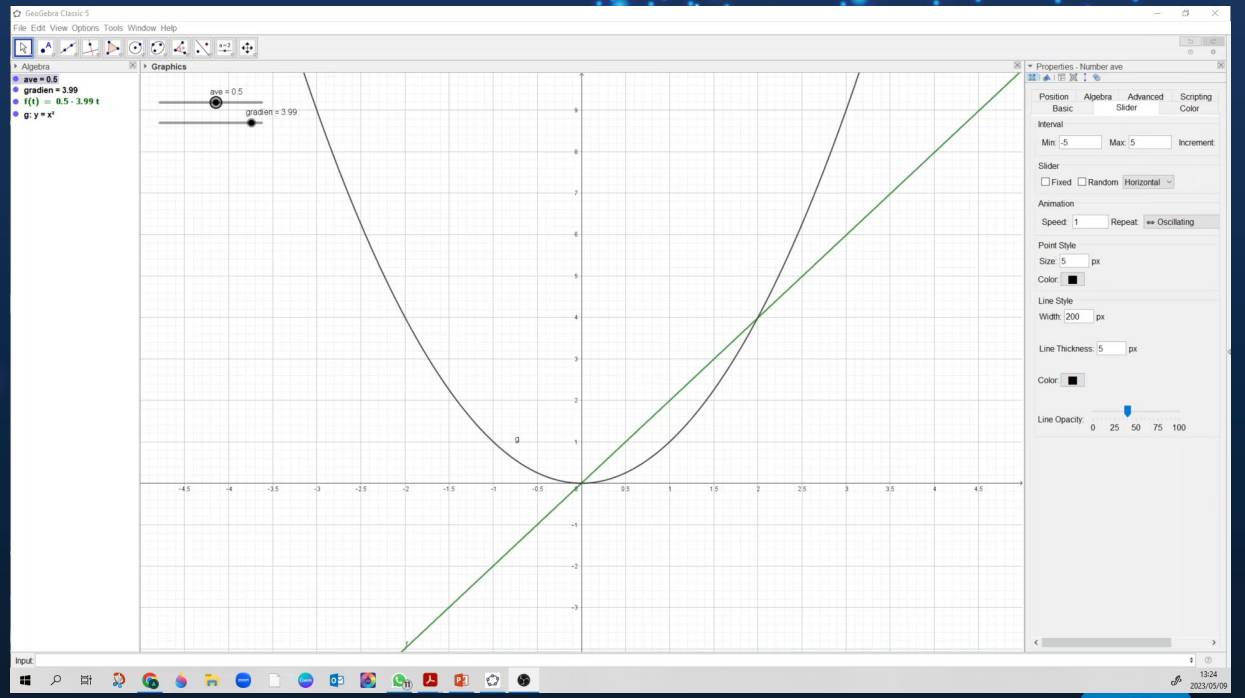
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

However, when dealing with curves the gradient changes at every point on the curve, therefore we need to work with the average gradient. The average gradient between two points is the gradient of a straight line drawn between the two points.



Average Gradient

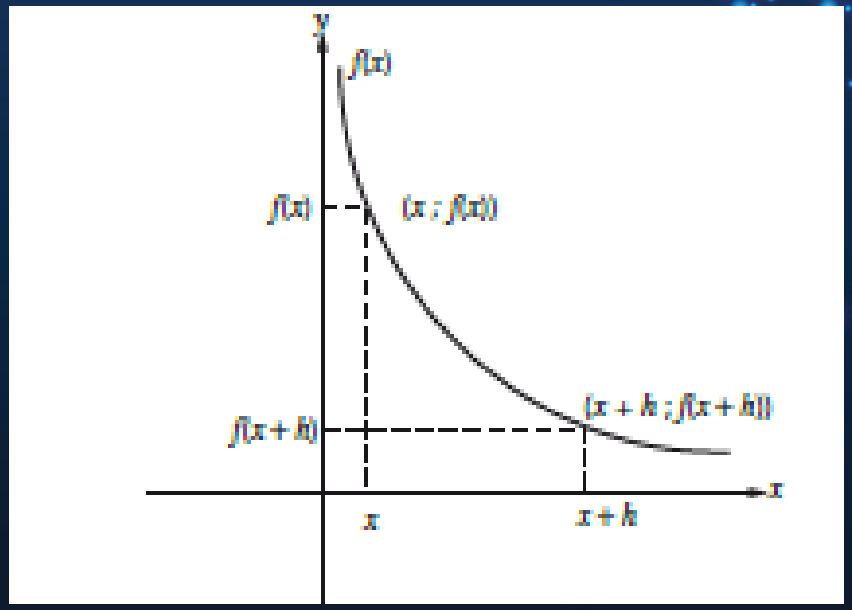
$$\frac{f(x+h)-f(x)}{h}$$

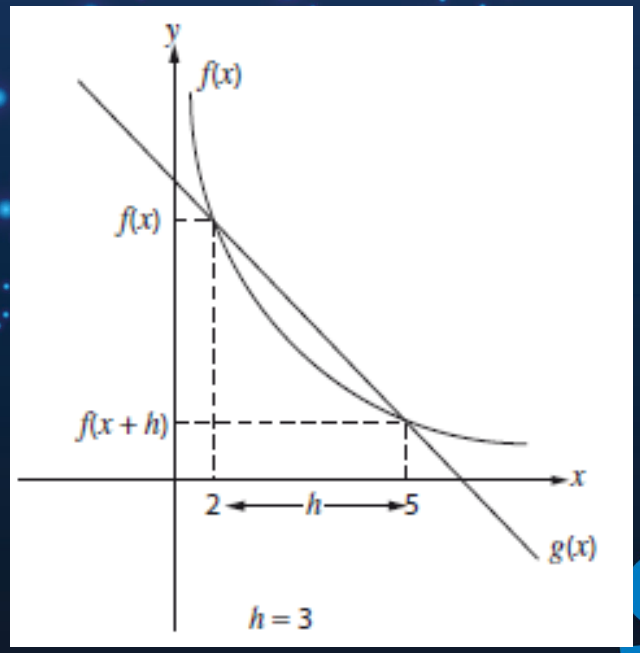


Average Gradient

$$\frac{f(x+h)-f(x)}{h}$$

Investigate the average gradient between 2 points. By calculation we can show that this is equivalent to:





Average Gradient

$$\frac{f(x+h)-f(x)}{h}$$

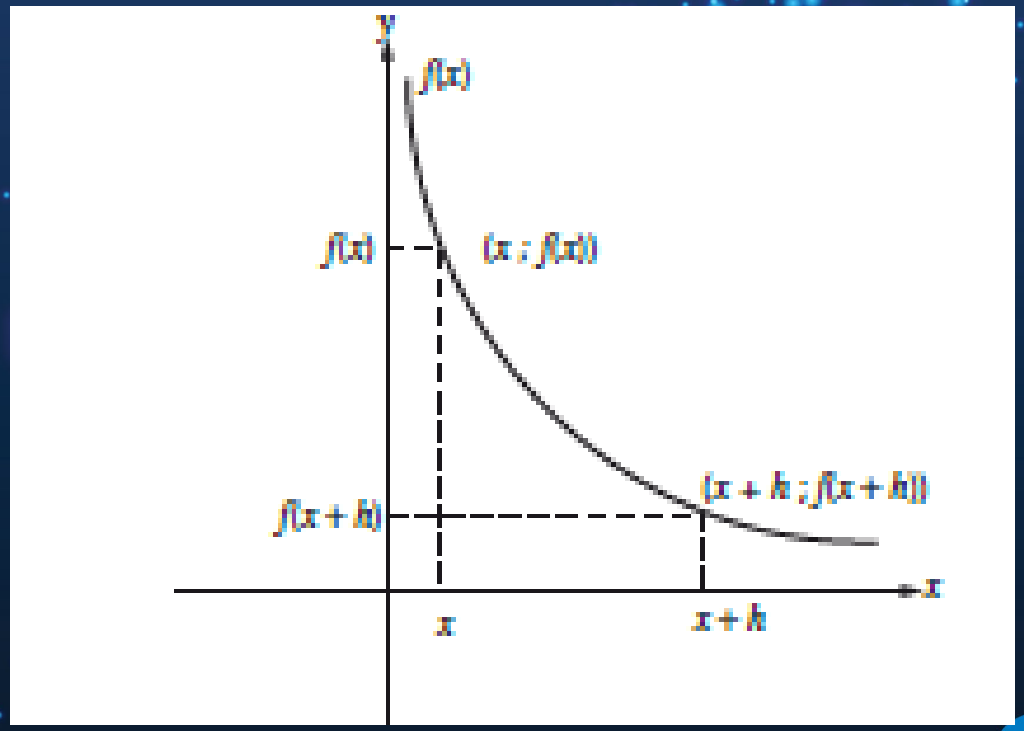
Can you also see that as the two points on the *x*-axis get closer together, the average gradient changes?
So as the two points get closer together (as *h* becomes 0) the average gradient between the two points becomes the actual gradient at the point.

However, *h* cannot equal 0 as that would give us a 0 denominator, so we must use limits.

The actual gradient at a point is called the derivative.

First Principle and Average Gradient

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



Rules for Differentiation and the language behind it

$D_x[f(x)]$ - the derivative of $f(x)$ with respect to x

$\frac{dy}{dx}$ - the derivative of y with respect to x

y' or f' - the derivative of y or $f(x)$

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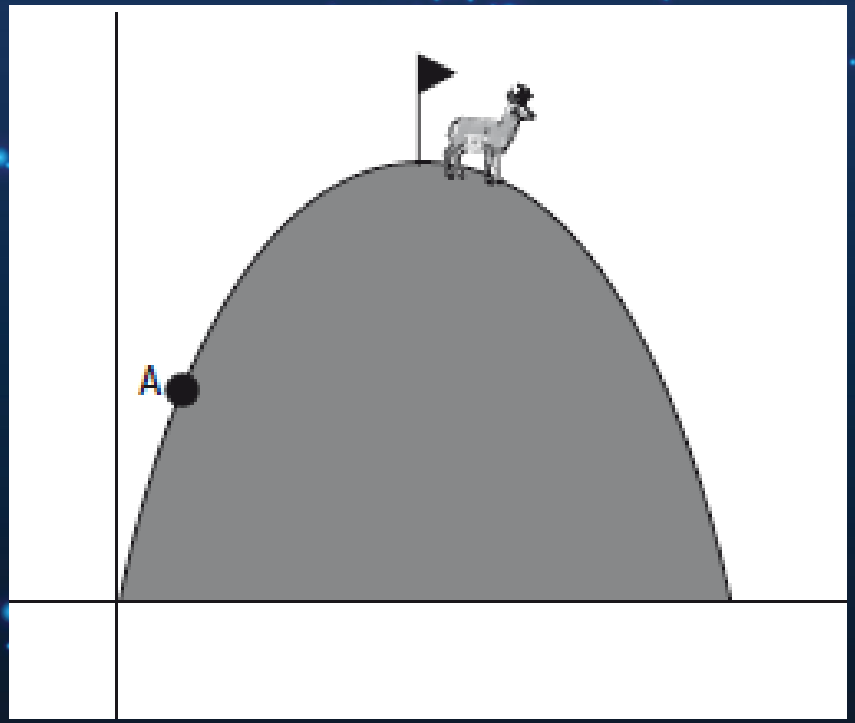
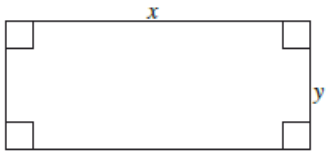
Application of Derivatives

Derivatives are useful in many aspects of life



Application of Derivatives

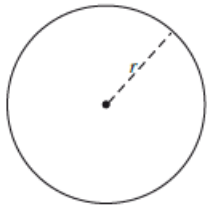
Derivatives are useful in many aspects of life to find maximum and or minimum of some quantity.



Rectangle

Perimeter = $2x + 2y$

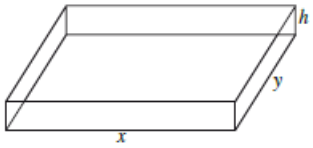
Area = xy



Circle

Perimeter (circumference) = $2\pi r$

Area = πr^2

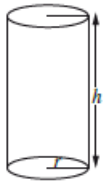


Box

Volume = xyh

Surface Area = $2xy + 2xh + 2yh$.....(closed)

Surface Area = $xy + 2xh + 2yh$.....(open)



Cylinder

Volume = $\pi r^2 h$

Surface Area = $2\pi rh + 2\pi r^2$.... (closed)

Surface Area = $2\pi rh + \pi r^2$... (open)

Application of Derivatives

- Science and Medicine
- Technology and Engineering
- Business and Economics
- History and Philosophy
- Arts and Athletics

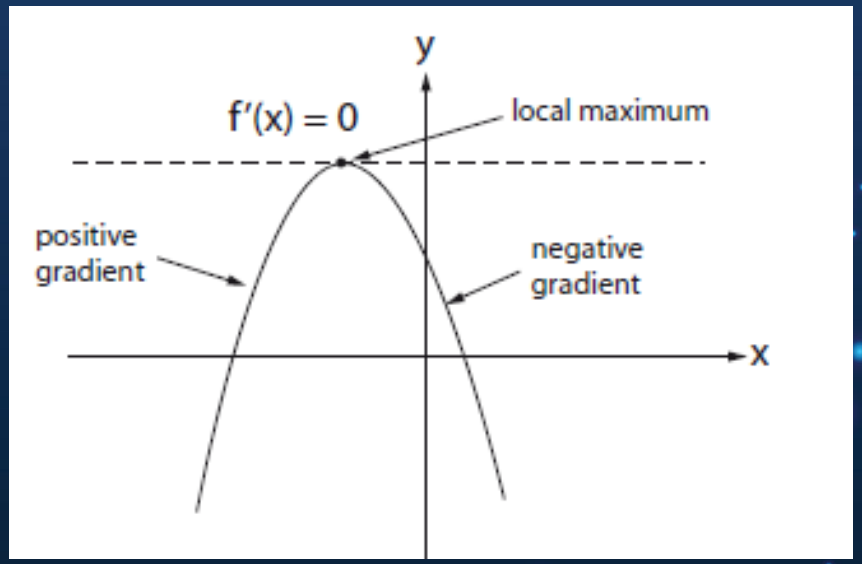
Cubic Functions

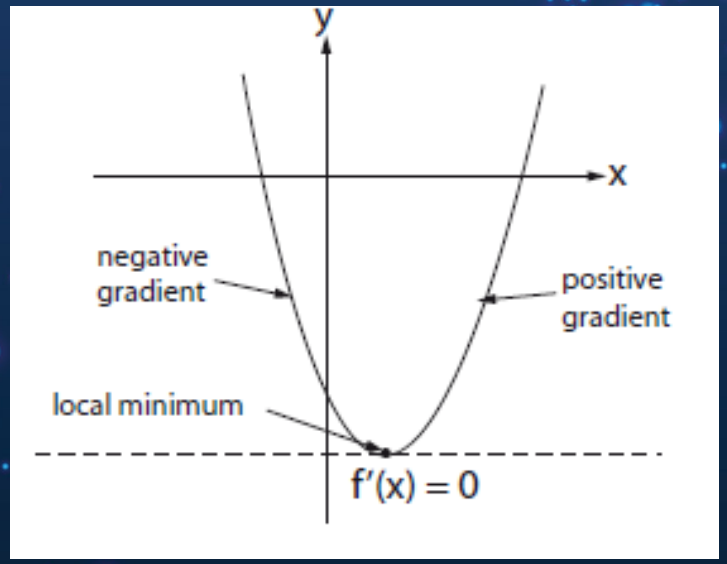
Important Concepts and Terminology

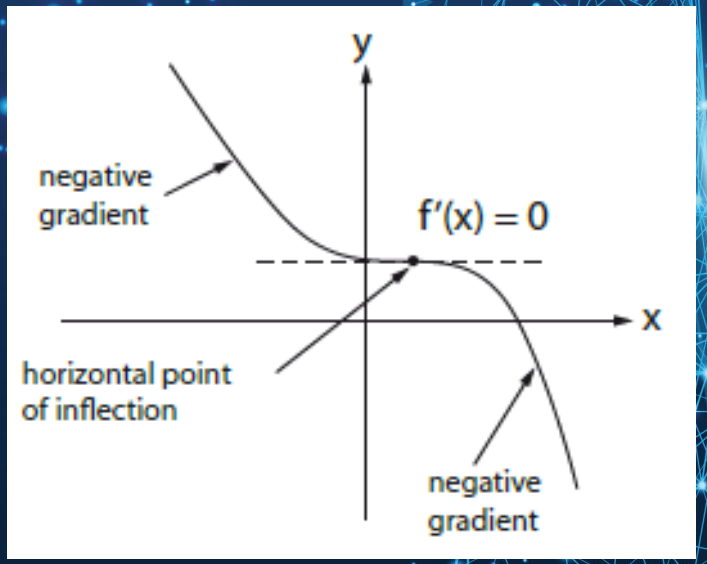
- Stationary points – zero gradient
- Local maximum
- Local minimum
- Negative gradient
- Positive gradient
- Point of Inflection

Cubic Functions

Important Concepts and Terminology



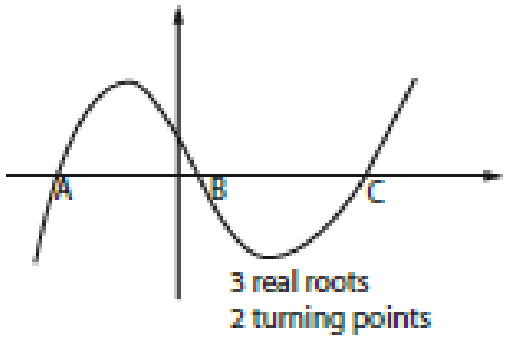




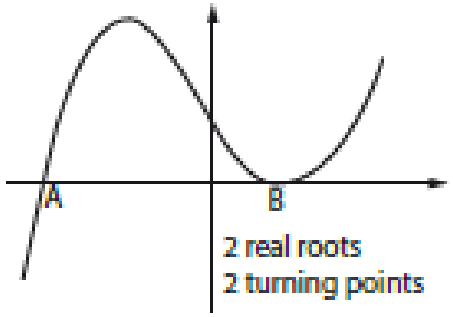
Cubic Functions

Important Concepts and Terminology

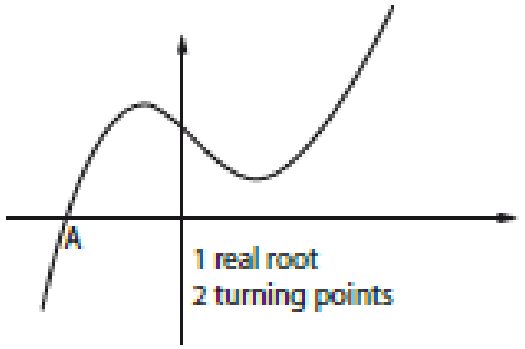
- TWO stationary points, in which case the possible graphs for a>0 are:



3 real roots
2 turning points

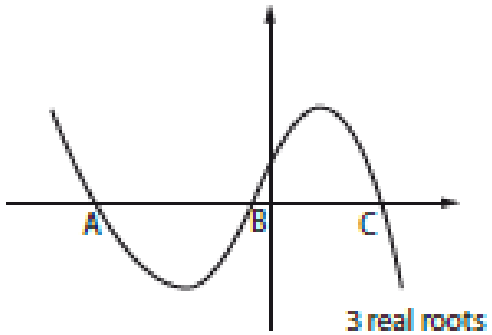


2 real roots
2 turning points

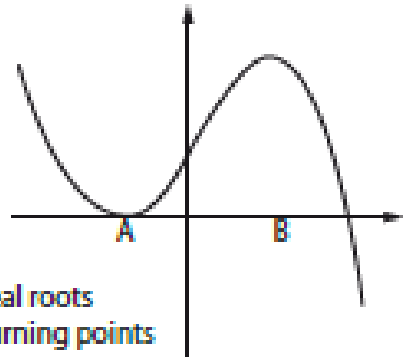


1 real root
2 turning points

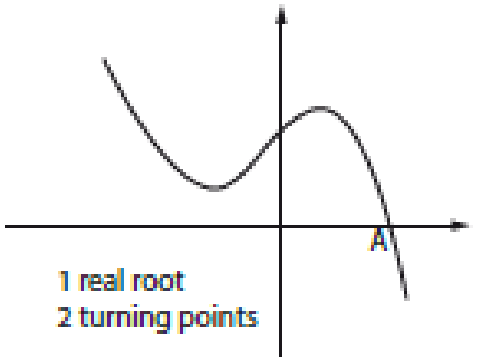
And for $a < 0$:



3 real roots
2 turning points



2 real roots
2 turning points

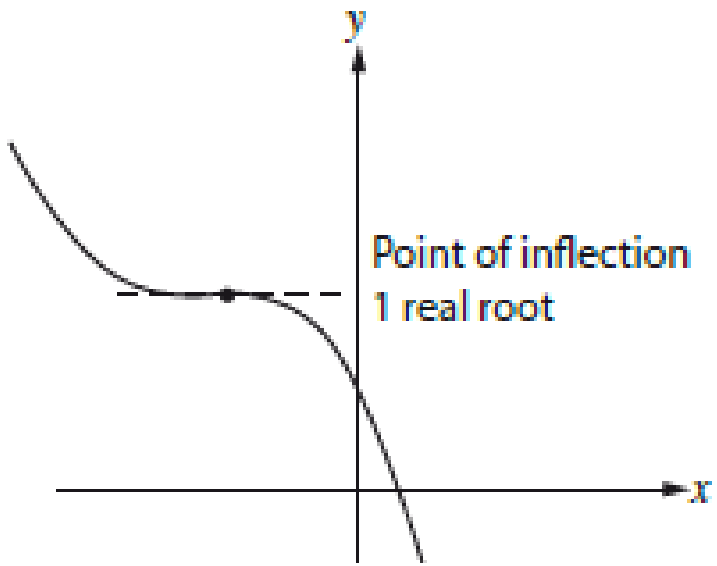


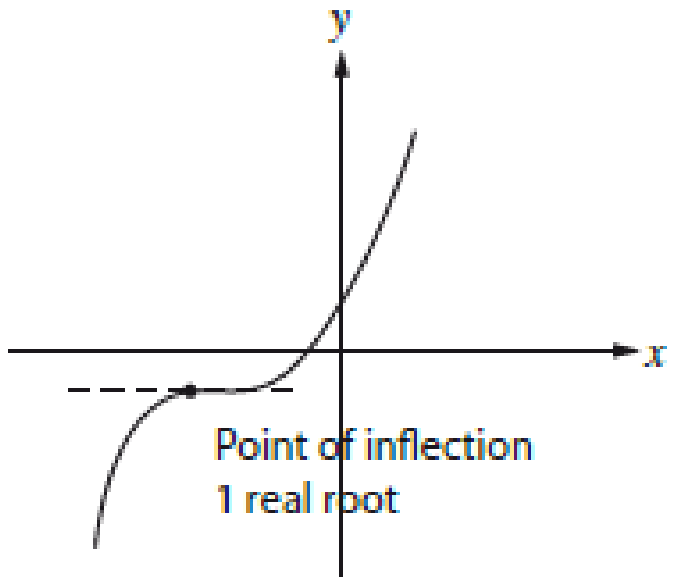
1 real root
2 turning points

Cubic Functions

Important Concepts and Terminology

- ONE stationary point, in which case the possible graphs are:





Cubic Functions

Stationary points – Minimum or maximum

$$f'(x) = 0$$

Point of Inflection

$$f''(x) = 0$$

Tangent first derivative will give the equation of the gradient of the tangent at a point.

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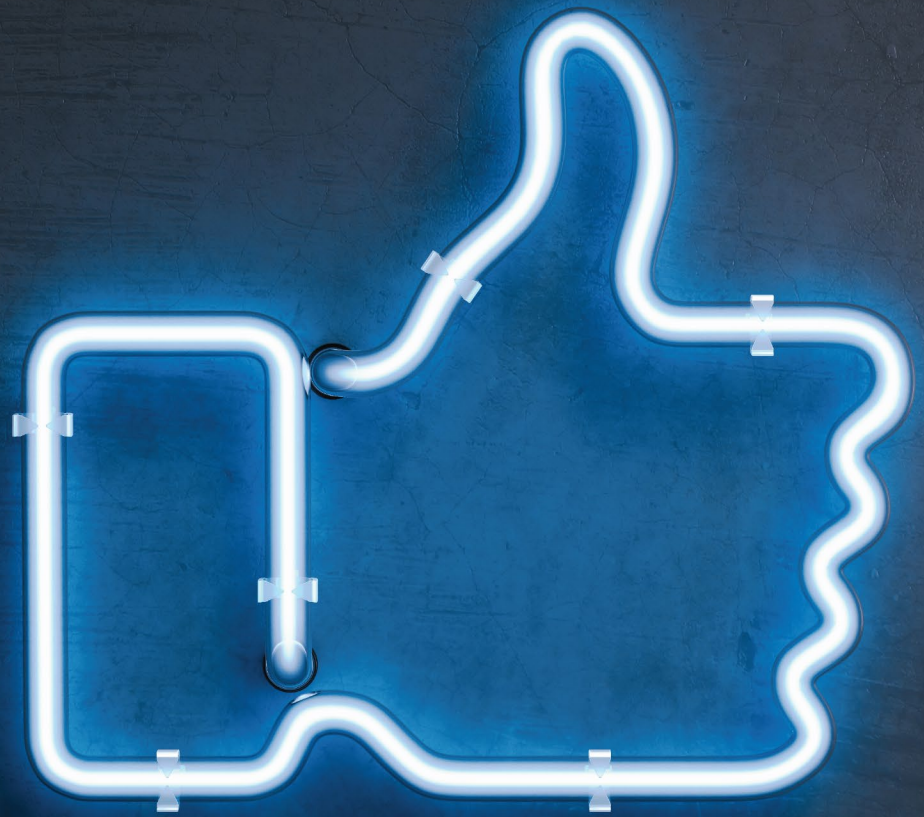

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