CHAPTER 2

TECHNICAL MATHEMATICS

The following report should be read in conjunction with the Technical Mathematics Paper 1 and Paper 2 question papers for the NSC November 2021 examination.

2.1 PERFORMANCE TRENDS (2018–2021)

In 2021, 13 403 learners sat for the Technical Mathematics examination. The number of candidates increased by 2 672 in 2021.

The performance of the candidates in 2021 shows a very significant increase when compared to the performance in 2020. The pass percentage at 30% (Level 2) improved from 32,4% in 2020 to 60,1% in 2021.

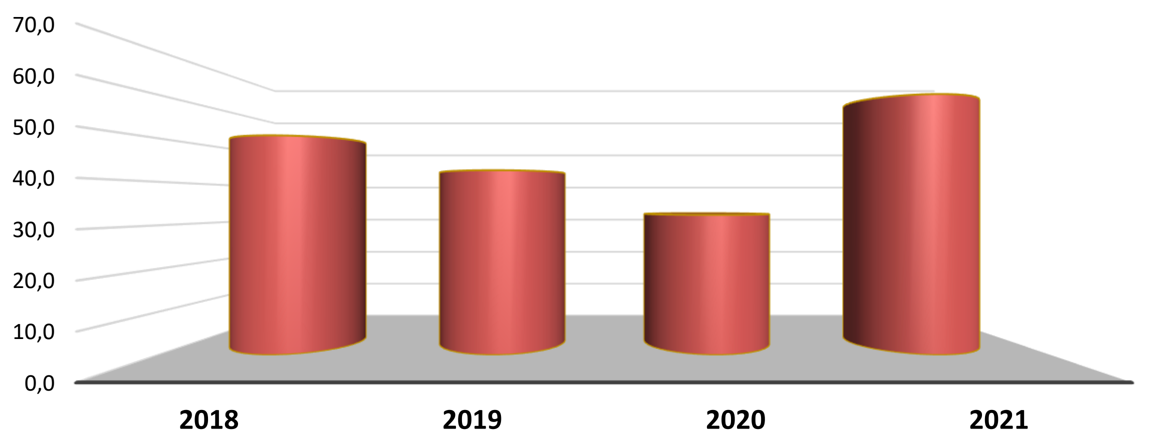
It was also very encouraging that 17,2% of candidates achieved over 50% this year in comparison to 8,4% of candidates doing so in 2020. This could be attributed to the inclusion of a PAT in the subject.

The percentage of distinctions (80% and above) increased marginally from 0,5% in 2020 to 0,6% in 2021. This translates to an increase in the distinctions from 54 in 2020 to 80 in 2021.

Table 2.1.1 Overall achievement rates in Technical Mathematics

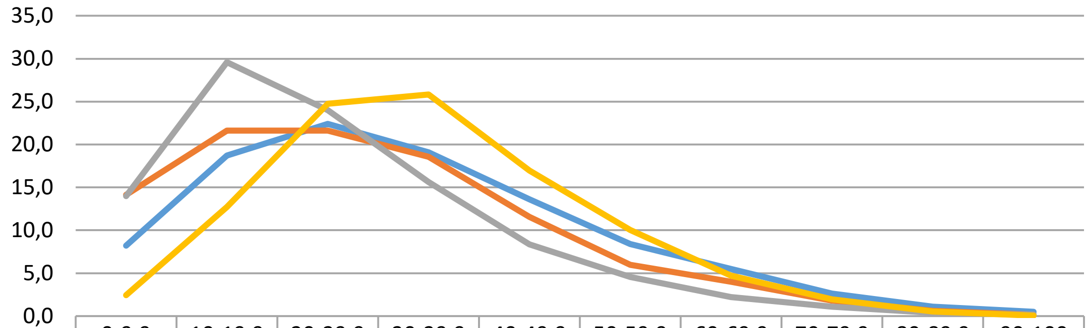
Year	No. wrote	No. achieved at 30% and above	% achieved at 30% and above
2018	10 025	5 078	50,7
2019	9 670	4 125	42,7
2020	10 731	3 476	32,4
2021	13 403	8 060	60,1

Graph 2.1.1 Overall achievement rates in Technical Mathematics (percentage)



	2018	2019	2020	2021
% achieved at 30% and above	50,7	42,7	32,4	60,1

Graph 2.1.2 Performance distribution curves in Technical Mathematics (percentage)



	0-9.9	10-19.9	20-29.9	30-39.9	40-49.9	50-59.9	60-69.9	70-79.9	80-89.9	90-100
2018	8,2	18,7	22,4	19,1	13,6	8,4	5,5	2,6	1,1	0,5
2019	14,1	21,6	21,6	18,6	11,5	6,0	4,0	1,8	0,6	0,2
2020	14,0	29,6	24,0	15,6	8,4	4,6	2,2	1,1	0,4	0,1
2021	2,4	12,7	24,7	25,8	17,0	10,0	4,7	1,9	0,5	0,1

Revision of work from earlier grades will play an integral part in improving performance in the subject. There is still room for improvement in the performance of candidates if the challenges surrounding mathematical skills, conceptual understanding and integration of topics are addressed.

As stipulated in the Technical Mathematics *CAPS*, 'Mathematical modelling is an important focal point of the curriculum' and that 'Real-life technical problems should be incorporated into all sections whenever appropriate'. Adequate attention should be paid to this comment. Performance will be further enhanced if candidates improve their ability to solve problems.

2.2 OVERVIEW OF CANDIDATES' PERFORMANCE IN PAPER 1

General comments

(a) Candidates performed very poorly in questions involving applications and modelling.

(b) Performance in topics taught in earlier grades was poor in comparison to performance in topics done in Grade 12. This was probably due to inadequate time being allocated for revision of work from the earlier grades.

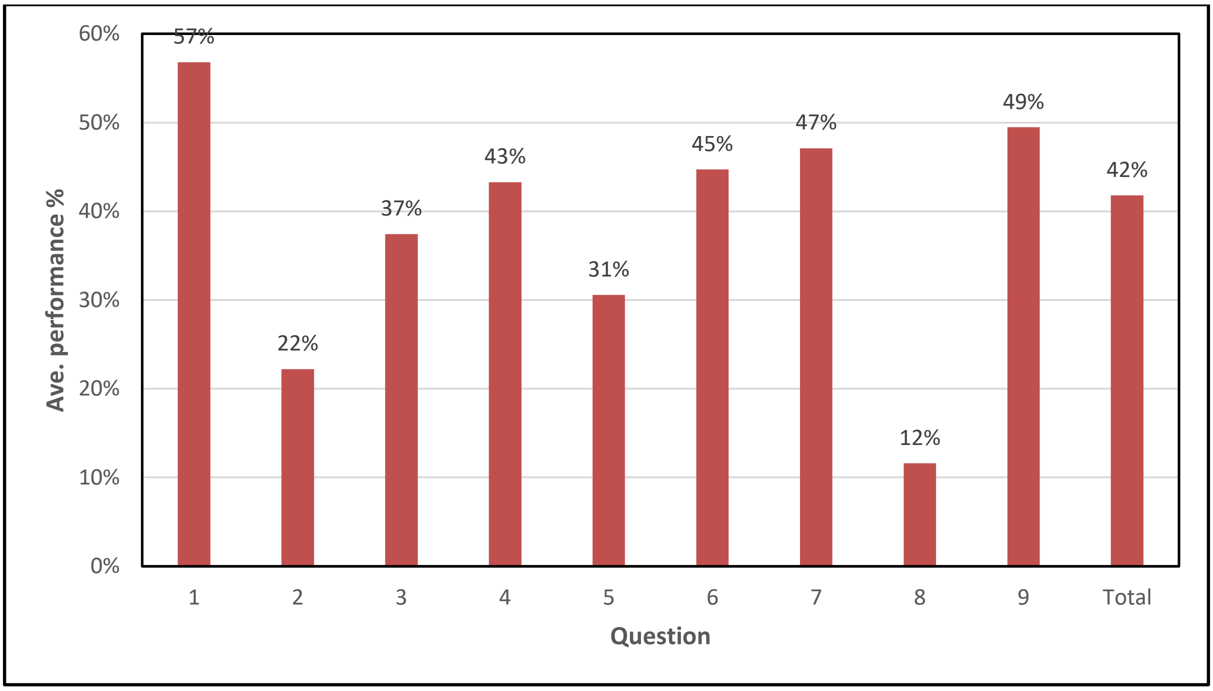
(c) Higher-order questions such as the interpretation of graphs as well as measurement and mensuration were either not answered or poorly answered. Questions in which topics were integrated proved to be challenging for many candidates.

(d) Candidates did not adhere to the instructions as stipulated in the question paper.

2.3 DIAGNOSTIC QUESTION ANALYSIS OF PAPER 1

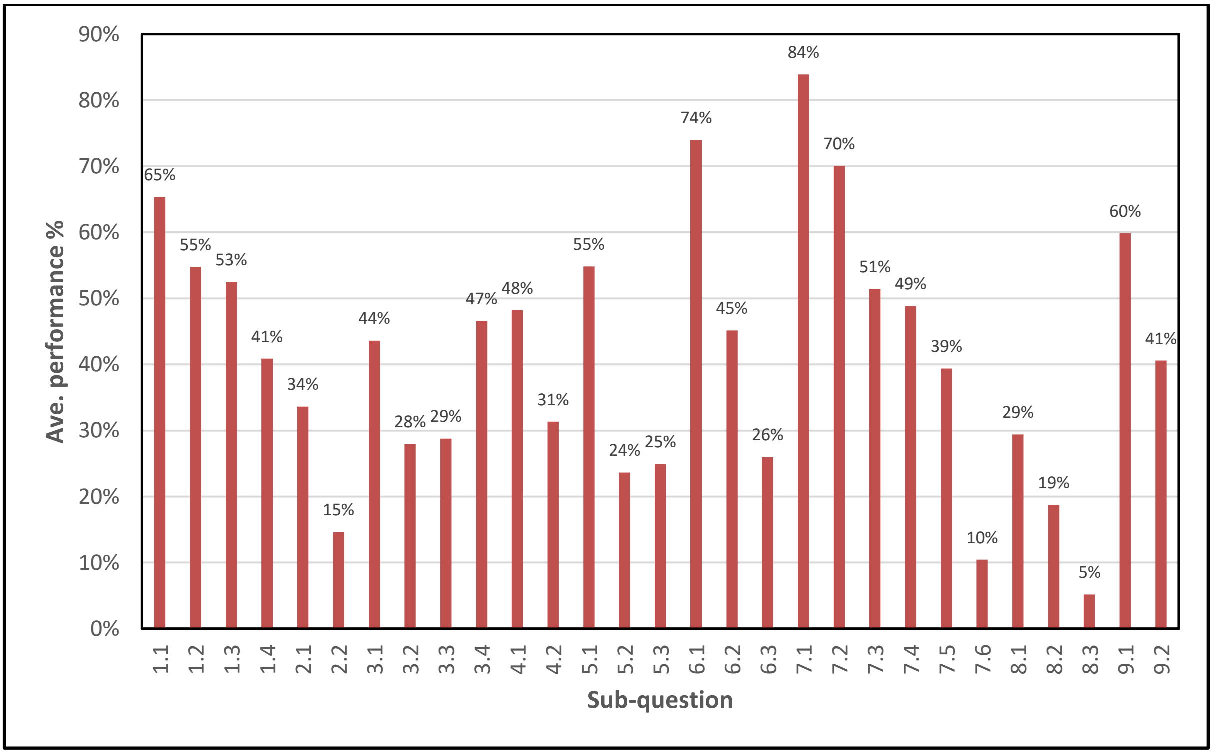
The following graph is based on data from a random sample of candidates. While this graph might not accurately reflect national averages, it is useful in assessing the relative degrees of challenge of each question as experienced by candidates.

Graph 2.3.1 Average performance per question in Paper 1



Q	Topic	Q	Topic
1	Equations, inequalities & binary numbers	6	Differential calculus (differentiation)
2	Nature of the roots of quadratic equations	7	Differential calculus (Cubic function)
3	Exponents & surds, logarithms & complex numbers	8	Differential calculus (Maxima and Minima)
4	Functions and graphs	9	Integration
5	Finance, growth & decay		

Graph 2.3.2 Average performance per subquestion in Paper 1



2.4 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION IN PAPER 1

QUESTION 1: EQUATIONS AND INEQUALITIES (ALGEBRA)

Common errors and misconceptions

(a) In Q1.1.1 candidates did not realise that the factors were given. Instead, they expanded and got to a standard quadratic form and then factorised again incorrectly or used a quadratic formula. Some failed to get $x = 0$.

(b) In Q1.1.2 some candidates failed to distribute the terms and copied the formula incorrectly. They displayed incompetency in using the calculator and rounding off to the required number of decimal places.

(c) Many candidates had difficulty in interpreting the inequalities in Q1.1.3. They showed limited understanding of the meaning of '*or*' and '*and'*. They failed to differentiate between greater than and less than.

(d) In Q1.2. candidates did not realise that x in the linear equation is the subject of the formula. They tried to create a third equation which had a fraction in it and found simplification a challenge later on in their answer.

(e) Candidates failed to remove the radical sign by squaring in Q1.3. Many candidates failed to simplify and make L the subject.

(f) In Q1.3.2 many candidates had difficulty in using a calculator after substitution and did not know what to do with 2π .

(g) In Q1.3 candidates were unable to square a binomial, simplify correctly and then determine the correct standard form.

(h) In Q1.4.1 many candidates failed to subtract the binary numbers and few omitted the base 2.

(i) In Q1.4.2 some candidates were unable to convert a binary number to a decimal number.

Suggestions for improvement

(a) Regular revision of topics done in earlier grades, e.g. factorisation, products, subject of the formula, solution of simultaneous equations and binary operations, is strongly suggested. Emphasise the writing of the correct notation and encourage the correct use of calculators.

(b) In teaching inequalities, graphical representation of the solution must be emphasised. Teachers should integrate Algebra with Functions so that learners have visual understanding of the region of the graph that is applicable to the inequality under consideration and explain the difference between "*or*" and "*and*" in the context of inequalities. Teachers should expose learners to different methods of solving inequality problems so that learners may choose the method best suited to solving the problem.

QUESTION 2: NATURE OF ROOTS

Common errors and misconceptions

(a) Some candidates failed to state the nature of roots satisfying the given conditions. In Q2.1.1, since the discriminant was negative, some concluded that roots are undefined.

(b) In Q2.1.2 candidates failed to describe the nature of roots but instead solved for *x*.

(c) In Q2.2 candidates omitted *q* when substituting in $\Delta = b^2 - 4ac$. They also used the '=' sign instead of '<' for non-real roots. Candidates accepted the positive value of *q* but rejected the negative value. This was incorrect.

Suggestions for improvement

(a) Teachers should demonstrate and explain to learners that the discriminant, $\Delta = b^2 - 4ac$, originates from the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. In other words, the quadratic formula could be written as $x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$. Teachers should emphasise that $\Delta = b^2 - 4ac$ is used to determine the nature of roots of the quadratic equation.

(b) By integrating the topics of functions and nature of roots, teachers can explain to learners that *non-real roots* and *undefined* are different concepts.

(c) Learners should be exposed to application involving the nature of roots. Examples should include proving problems about nature of roots applicable to given conditions.

QUESTION 3: EXPONENTS, SURDS, LOGARITHMS AND COMPLEX NUMBERS

Common errors and misconceptions

(a) In Q3.1.1 many candidates could not write 81 in exponential form.

(b) Some candidates failed to apply logarithmic properties in Q3.1.2.

(c) Candidates had difficulty in multiplying surds in Q3.1.3. This would have enabled them to simplify the expression. Also, they were unable to convert from surd form to exponential form and apply exponential rules.

(d) In Q3.2 some candidates failed to apply logarithmic properties and convert the log form into an exponential form. They confused $2 + log_3 x$ with $2log_3 x$ leading to $log_3 x^2$

(e) Many candidates were unable to differentiate between the modulus and the real part of the complex number, *p*, in Q3.3.1. They showed a lack of understanding that $p^2 = \sqrt{4}$ has 2 roots. They only wrote 2 as the value of *p*, disregarding the given condition for *p*.

(f) In Q3.3.2 candidates failed to get the correct angle within the given interval. They gave θ in terms of an acute angle.

(g) In Q3.4 some candidates did not apply the distributive property. Furthermore, they did not equate the real part to a real part and imaginary part to an imaginary part.

Suggestions for improvement

(a) Revision of all exponential, surd and logarithmic laws done in earlier grades by learners is strongly suggested in Grade 12.

(b) Teachers need to strengthen the concept of factors and products and reinforce the method of converting from exponential form to surd and/or logarithmic forms and vice versa.

(c) The use of calculators to check the correctness of their solutions should be emphasised to learners.

(d) Teachers should expose learners to different types of problems involving complex numbers and ensure that learners adhere to the given instructions.

QUESTION 4: FUNCTIONS

Common errors and misconceptions

(a) In Q4.1.1(a), (c) and (d) some candidates had difficulty differentiating between the intercepts and the turning point. They failed to factorise the quadratic equation and opted to use the quadratic formula. They swopped the values of *a*, *b* and *c* when substituting into the formula.

(b) Many candidates failed to write the equation of the asymptote in Q4.1.1(b). Furthermore, they were unable to differentiate between a function value, asymptote and a parameter. They wrote the equation of the horizontal asymptote as $h(x)=5$ and some wrote it as $q=5$.

(c) In Q4.1.2 some candidates could not sketch the graphs.

(d) Some candidates struggled with interpreting Q4.1.3. They were unable to identify and substitute the parameters correctly. They substituted the *y*-value of the given point for *q*, which is an asymptote, and 0 for *p*(*x*).

(e) In Q4.2.1, Q4.2.2 and Q4.2.3 candidates were unable to identify the characteristics of graphs. They failed to write the equation of the semi-circle correctly. Some substituted the value of *r* and *x*. This was incorrect.

(f) In Q4.2.4 candidates had difficulty with the interpretation of graphs.

Suggestions for improvement

(a) Characteristics of graphs and the effect of parameters should be thoroughly demonstrated and explained to learners.

(b) Teachers should point out to learners that when a question asks the equation of the asymptote, it should be presented as *y* =…. or *x* =…, depending on which asymptotes are required.

(c) When learners do not recognise the correct shape of the graph, they should be advised to use several points from a table to draw functions.

(d) Teachers should explain the meaning of inequalities and the definition and correct notation of the domain and range of a function.

(e) Teachers should expose learners to different ways of determining the equations of the graphs.

(f) Teachers should expose learners to different questions involving two graphs on the same system of axes. This should not be limited to drawing graphs but also the interpretation of graphs as well.

QUESTION 5: FINANCE, GROWTH AND DECAY

Common errors and misconceptions

(a) Some candidates failed to express the fraction as a percentage in Q5.1.1.

(b) In Q5.1.2 candidates failed to use the correct formula.

(c) Many candidates had difficulty interpreting Q5.2. Some candidates did not realise that they had to use the reducing-balance depreciation formula. Some candidates interchanged the values of A and P. This was incorrect. Some candidates used the method of verification by using the compound growth rate. This was not acceptable. Many candidates did not realise that the final answer was $n > 5{,}67$ while they wrote $n = 5{,}67$ instead.

(d) Some candidates used an incorrect formula in Q5.3.1. They failed to interpret the question correctly and used R15 000 instead of R25 000 as the value for P.

(e) In Q5.3.2 many candidates showed a lack of understanding of the different compounding periods. Some candidates failed to use the correct formula. Most candidates did not realise that R6 823,54 had to be added to the value of the investment after 27 months.

(f) Many candidates failed to use the calculator correctly and they did not make a conclusion.

Suggestions for improvement

(a) Revision of work done in earlier grades, among others, fractions, percentages, interest, hire purchase, inflation and other real-life problems, is strongly suggested.

(b) Learners should identify the correct formula from the information sheet given with the question paper.

(c) Teachers need to explain to learners that in all formulae, P represents the initial value. In the case of a population, P represents the initial number and A represents the final number of species in the situation. Teachers need to emphasise that in scenarios that involve depreciation, the value of P will be greater than the value of A.

(d) Teachers should also demonstrate to learners how to change the subject of the formula. It is advised that learners first substitute values in a formula and then change the subject of the formula.

(e) All compounding periods (annually, quarterly, monthly, semi-annually/half-yearly and even daily) should be taught to learners. The use of timelines in order to better understand a complex problem involving several investments, deposits and withdrawals, is strongly advised.

(f) Learners should be competent in using a calculator.

(g) A good understanding of Financial Mathematics is best developed through practice.

QUESTION 6: CALCULUS

Common errors and misconceptions

(a) In determining the derivative using first principles in Q 6.1, some candidates:
- incorrectly used the rules for differentiation instead.
- failed to copy the definition correctly from the information sheet.
- used incorrect notation by omitting $\lim_{h \to 0}$ or by placing the = sign in the incorrect position $\lim_{h \to 0} = \frac{f(x+h)-f(x)}{h}$ or by using the incorrect notation $f(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.
- failed to substitute correctly into the formula. Some omitted brackets when substituting for $f(x)$.

(b) In Q6.2.1 candidates failed to differentiate the expression correctly because it had 2 variables, *x* and *p*. They did not realise that *p* should be treated as a constant. They incorrectly wrote the derivative of p^3x^2 as $3p^2 2x$ or $6p^2x$.

(c) Candidates had difficulty simplifying a fraction that contained exponents.

(d) In Q6.2.2 most candidates failed to write the surd in exponential form, i.e., $\sqrt[3]{x^2}$ as $x^{\frac{2}{3}}$

(e) A few candidates integrated the given expressions instead of differentiating between them.

(f) In Q6.3.1 many candidates failed to interpret the question correctly. They did not recall that when two lines are perpendicular to each other, then the product of their gradients is equal to –1. This was possibly due to the fact that this concept is mainly assessed in Paper 2.

(g) In Q6.3.2 many candidates demonstrated limited knowledge of the relationship between the gradient of the tangent and the derivative of the curve at the point of contact. They equated the derivative to 0 and solved for *x* instead of equating the derivative of the curve to the gradient of the tangent.

(h) In Q6.3.3 candidates used an incorrect formula for average gradient. They incorrectly wrote Average grad. $= \frac{x_2 - x_1}{y_2 - y_1}$. Some candidiates failed to get the correct values for *g*(-2) and *g*(3) because of poor simplification skills.

Suggestions for improvement

(a) Learners should copy the formula for first principles correctly from the information sheet attached to the question paper.

(b) Teachers should emphasise that when using first principles, correct notation should be used. The $\lim_{h \to 0}$ must be written down in each step and should only be left out when writing the final step, i.e. once the learner has substituted the value of *h*.

(c) Simplification of expressions involving algebraic fractions should be revised with learners.

(d) Learners should be exposed to various notations used in differentiation. The notations $f^{/}(x)$ if $f(x) = x^n$, $\frac{dy}{dx}$ if $y = x^n$, $\frac{d}{dx}\left(x^n\right)$ and $D_x\left(x^n\right)$ all have the same meaning. Teachers should thoroughly explain the difference between the Derivative and an Integral of a function and demonstrate the difference of each by calculating the derivative and integral of the same function.

(e) Differentiation involves working with the exponent. Revision of exponential and surd laws is encouraged before starting with the topic of differentiation.

(f) Teachers should define the derivative in relation to gradient at a point on a curve or gradient of a tangent and demonstrate this relationship using dynamic mathematical software.

QUESTION 7: CUBIC FUNCTION

Common errors and misconceptions

(a) Few candidates failed to calculate the *y*-intercept correctly in Q7.1

(b) In Q7.2 some candidates demonstrated limited understanding of the Factor Theorem.

(c) Some candidates did not realise that the information in Q7.2 should be used in Q7.3. They used a quadratic formula to solve for *x* in the cubic function. Some candidates calculated the coordinates of the turning points.

(d) In Q7.4 some candidates calculated the *x*-intercepts instead of the coordinates of the turning points. Other candidates used the method of finding the turning point of a quadratic function without realising that this method was not applicable to cubic functions. Most candidates did not equate the derivative function to zero. Many candidates managed to calculate the two *x*-coordinates of the turning points correctly but failed to substitute these values into the original function when determining the corresponding *y*-coordinates.

(e) Many candidates had difficulty drawing the graph as this question was dependent on the responses from Q7.1, Q7.2, Q7.3 and Q7.4. A few candidates did not attach their answer sheet nor did they draw the graph in their answer books.

(f) Many candidates were unable to identify the interval in which the graph was decreasing. They made errors with the notation and/or critical values.

Suggestions for improvement

(a) Teachers should explain the characteristics of graphs and demonstrate them by examples and illustrations.

(b) Learners should read the given information carefully and respond to the questions asked.

(c) Teachers should expose learners to various forms of graphical representations and all aspects of the functions, including sketching and interpretation of the graphs.

(d) The concept of the derivative function and the turning point should be explained in detail. Teachers need to emphasise that the derivative function is equal to zero at the turning points. Teachers should indicate to learners that calculating the *x*-coordinate of the turning point using $x = -\frac{b}{2a}$ only applies to quadratic functions.

(e) Teachers should explain the concept of minima and maxima and demonstrate to learners where the graph is increasing, decreasing and stationary by means of diagrams. Teachers should use a variety of available software to illustrate where the graph is decreasing, increasing or stationary. This should assist in the interpretation of functions. Software such as Geometry Sketch Pad, Graph and GeoGebra are useful tools to demonstrate these concepts.

QUESTION 8: APPLICATION OF CALCULUS

Common errors and misconceptions

(a) In Q8.1 many candidates used an incorrect formula for volume. Others failed to substitute correctly into the formula for volume and then make *h* the subject. They manipulated the given equation and solved for *x*.

(b) Many candidates were unable to determine the expression for the surface area of the container in Q8.2.

(c) In Q8.3 many candidates did not realise they needed to use the given total surface area in Q8.2 to solve for *x*. They wrote the height in terms of *x* in Q8.1.1. Some candidates failed to differentiate with respect to *x*. Further, they did not find the cube root when solving for *x*. After calculating the value of *x*, some candidates did not realise that they still needed to calculate the value of *h*.

Suggestions for improvement

(a) Learners should be given tasks where they are required to manipulate different formulae involving prisms, solids, cones and pyramids etc. Teachers should demonstrate how these problems relate to the real-life context.

(b) Learners should be exposed to examples involving contextual applications.

(c) Learners need to be taught that when the question asks to 'Show/Prove that …', it means calculate (justify by means of mathematically correct steps) what is given and the final answer reached must match what is stated in the question.

(d) Teachers should explain to learners that the word 'hence' means to use the information obtained in the previous question to solve the question at hand.

(e) Teachers should explain the concept of minima and maxima in the context of optimisation. In optimisation, teachers need to explain that the first step is to find the derivative, equate the derivative to 0 and then factorise or use the quadratic formula to calculate the value of x and then substitute the value of x in the original function to find the dimension to be optimised.

QUESTION 9: INTEGRATION

Common errors and misconceptions

(a) In Q9.1.1 some candidates failed to remove the bracket before integrating the expression. Only a few candidates removed the bracket incorrectly.

(b) Many candidates did not realise that the expression in Q9.1.2 is similar to the one given in the information sheet. Some candidates omitted C when determining the indefinite integral.

(c) In Q9.2 candidates failed to write the correct notation for area when using integrals. Some candidates differentiated the expression. Candidates calculated the area of the shaded part even though it was given. Very few candidates substituted the boundaries in the given function without first integrating. Many candidates failed to equate the area of the shaded part to the area bounded by the curve and the x-axis between $x = k$ and $x = 4$.

Suggestions for improvement

(a) Simplification of expressions covered in earlier grades should be revised.

(b) Teachers should explain to learners that when determining indefinite integrals, the constant C must always be added. The use of the correct integral notation should be emphasised.

(c) In the teaching of integration, learners should be taught to include the lower and upper boundaries when setting up the area notation for definite integrals. The use of brackets should be emphasised when substituting negative values.

(d) The calculation of the area bounded by a function and the x-axis should be demonstrated so that learners can observe how the value of the constant influences the area.

(e) Learners should be exposed to a variety of problems involving integration to enhance their understanding of the concept of integration.

2.5 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION IN PAPER 2

(a) Candidates performed relatively well in Q1. This question was based on Analytical Geometry. Candidates performed extremely well in questions involving gradient and the equation of the straight line, which are concepts covered in Grade 10.

(b) Candidates performed poorly in Q3, Q4, Q6, Q7, Q8 and Q9 with Q9 being the worst answered question.

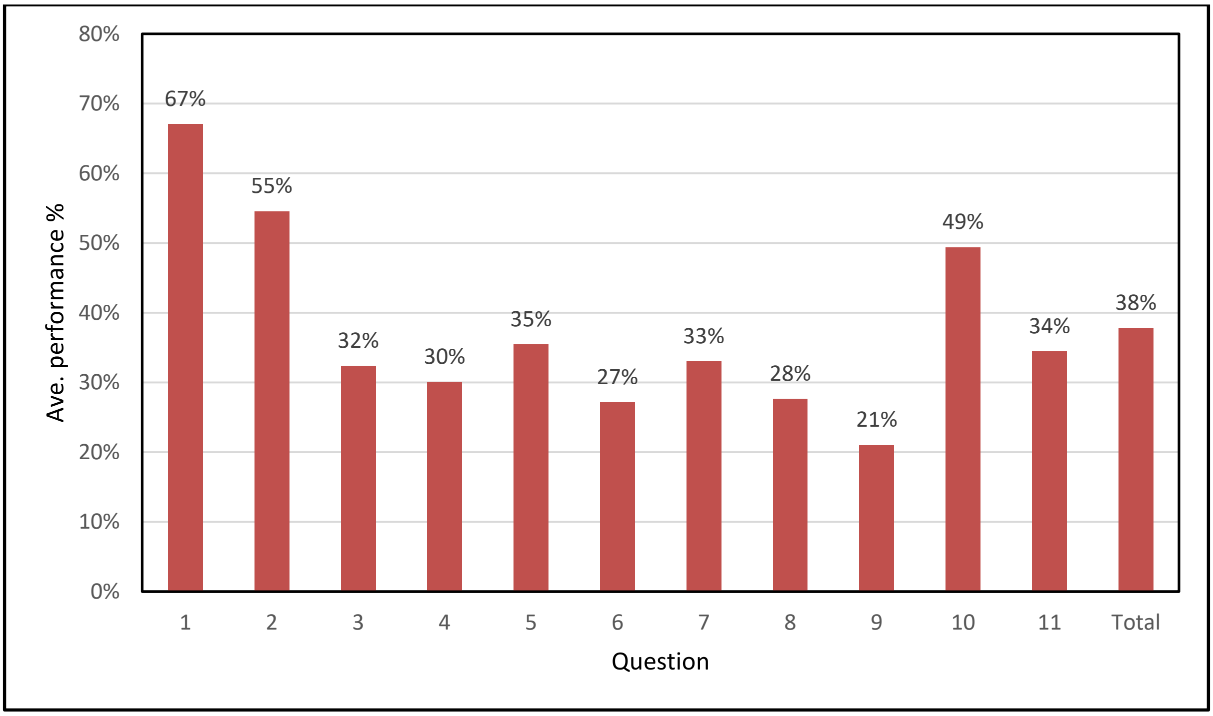
(c) Candidates did not adhere to the instructions stipulated in the question paper.

(d) Many candidates did not attempt the higher-order questions.

2.6 DIAGNOSTIC QUESTION ANALYSIS OF PAPER 2

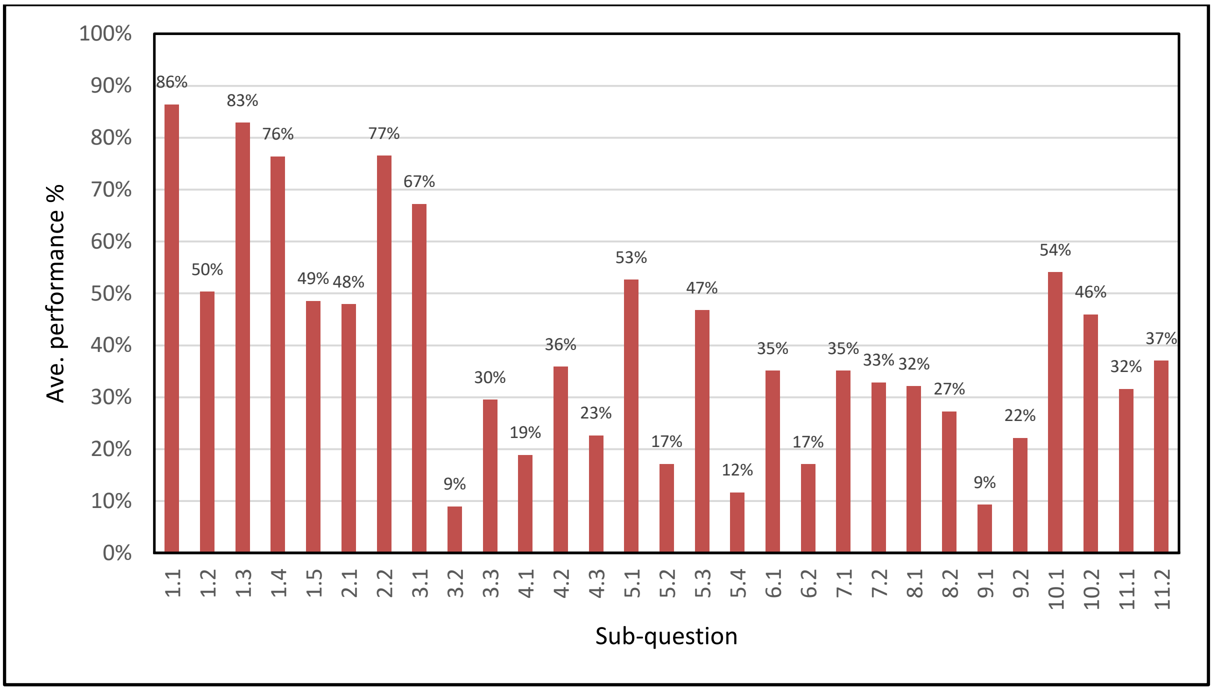
The following graph is based on data from a random sample of candidates. While this graph might not accurately reflect national averages, it is useful to assess the relative degrees of challenge of each question as experienced by candidates.

Graph 2.6.1 Average performance per question in Paper 2



Q	Topics	Q	Topics
1	Analytical Geometry - Lines	7	Euclidean Geometry - Circle with tangents
2	Analytical Geometry - Circle; Tangents; Ellipse	8	Euclidean Geometry - Circle - Cyclic quads
3	Trigonometry - General ratio's and equations	9	Euclidean Geometry - Proportionality
4	Trigonometry - Identities	10	Mensuration- Angular velocity
5	Trigonometry - Functions and graphs	11	Mensuration- Area and Volume
6	Trigonometry - 2D and 3D Applications		

Graph 2.6.2 Average performance per subquestion in Paper 2



2.7 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION IN PAPER 2

QUESTION 1: ANALYTICAL GEOMETRY

Common errors and misconceptions

(a) When answering Q1.2, candidates did not know that the reference angle was a positive acute angle. Some did not realise that the required angle was an obtuse angle.

(b) In Q 1.4 candidates still made incorrect substitution into distance formula. Some candidates used irrelevant points in their calculation.

(c) When answering Q1.5, candidates used the gradient calculated in Q1.1 instead of the negative reciprocal of the gradient calculated in Q1.1. These candidates confused the relationship of the gradients of perpendicular lines with that of parallel lines.

Suggestions for improvement

(a) Learners should be taught that reference angle is an angle in the first quadrant, hence it should not exceed 90 º.

(b) When given the midpoint of a straight line and the coordinates of an end point, learners should not swap the coordinates when substituting into the midpoint formula. Teachers should emphasise correct substitution of *x* and *y* values.

(c) Teachers should explain the difference in calculating the ratio of an angle and the size of an angle.

(d) Teachers should emphasise the conditions for lines to be parallel and perpendicular. Learners should know the difference between the length of a line and its gradient.

QUESTION 2: ANALYTICAL GEOMETRY

Common errors and misconceptions

(a) Some candidates calculated the value of *r* correctly but then did not write the equation of the circle.

(b) Most candidates did not realise that they had to swop the signs of the coordinates of B in order to obtain the coordinates A.

(c) In Q2.1.3 candidates were unable to answer questions that required them to "Show that …". Most assumed the value of *t* and used it to calculate the gradient of BC. This was not accepted as correct.

(d) When answering Q2.1.4, some candidates did not use the value given in Q2.1.3 and the coordinates of B. This meant that the answer did not respond to the question asked.

(e) Some candidates could not calculate the value of *t* because they could not determine the equation in Q2.1.4.

(f) In Q2.2 some candidates did not use the square roots of 49 and 4 to determine the intercepts, hence their sketch was incorrect.

Suggestions for improvement

(a) Learners should be trained to write down given information on the sketch.

(b) Learners should be taught the rules for the various transformations of a point on a circle. It will help them to determine the coordinates of the translated points on the circle with ease.

(c) Teachers should teach learners that 'show questions' require learners to use information available to arrive at what they are required to show.

(d) Teachers should always integrate different topics in their teaching. The concept of the gradient of the tangent at a point requires the application of Euclidean Geometry.

(e) Teachers should inform learners that they should use as given information that which they were expected to show in a previous question when answering subsequent questions.

(f) Learners should be taught to how to determine the *x*- or *y*-coordinate of a point if they are given the coordinates of another point and the gradient of the line passing through these two points.

(g) Learners should know the standard form of the ellipse and that when drawing, the values of *a* and *b,* and not a^2 and b^2, are used

QUESTION 3: TRIGONOMETRY

Common errors and misconceptions

(a) In Q3.1.1 some candidates misplaced the brackets in their calculation. They arrived at the incorrect answer of $\cos 15,8° - \cos(2)74,1° = -73,09$.

(b) The coefficient of 3 in Q3.1.2 became a problem for many candidates as they made the following mistake in their final answer $\frac{1}{3\sin 87,95°}$ instead of $\frac{3}{\sin 87,95°}$.

(c) When answering Q3.2.1 many candidates did not draw a diagram or failed to use the quadratic identity. Many candidates did not understand the meaning of *write in terms of.*

(d) In Q3.2.3 many candidates failed to reduce 317° to either 43° or 47°.

(e) In Q3.3 many candidates stopped after calculating the reference angle and did not consider the interval for which they had to calculate the angle.

Suggestions for improvement

(a) Learners should practise substituting correctly into an expression. They should also be aware that when calculating the value of a trigonometric ratio, the calculator assumes that the angle is within the set of brackets. Therefore, the correct use of brackets is important when calculating the value of trigonometric ratios, e.g. $\cos(2)74,1$ and $\cos[(2)74,1]$ will give two different answers on the calculator.

(b) Learners should know that the coefficient of the reciprocal ratio is not affected when it is changed to trigonometric ratio. Teachers should demonstrate how to calculate the values of reciprocal trigonometric ratios using a calculator.

(c) Teachers should emphasise the importance of drawing a diagram for the given ratio in the form of a right-angled triangle in the correct quadrant. This should allow learners to determine the third side and thereby enable them to write the values of all the trigonometric ratios and reciprocal trigonometric ratios for the given angle.

(d) Learners should be taught reduction formulae in order to reduce angles to their equivalent acute angle.

(e) Learners should practise algebraic manipulation skills and should be competent when performing these in calculating the magnitude of an angle for a given ratio.

QUESTION 4: TRIGONOMETRY

Common errors and misconceptions

(a) Candidates did not recognise that the response to Q4.1 was a trigonometric identity.

(b) Applying reduction formulae and simplifying trigonometric expressions using identities still proved to be a challenge in Q4.2.

(c) In Q4.3 some candidates were unable to reduce angles given as radians.

Suggestions for improvement

(a) Learners should be able to recognise identities where different symbols are used to represent the angle. It is advisable that the teacher uses a number of different symbols to represent angles in classwork.

(b) Teachers should expose learners to a variety of problems that require reduction formulae and the use of different prescribed identities in simplifying trigonometric expressions.

(c) Teachers should include reduction of angles given as radians when teaching reduction formulae.

QUESTION 5: TRIGONOMETRIC FUNCTIONS

Common errors and misconceptions

(a) Most candidates managed to draw the required cosine graph in Q5.1. Some candidates were unable to draw the graph of $y = \sin x + 1$ correctly. They did not translate the graph of $y = \sin x$ one unit upwards.

(b) Many candidates were unable to determine the period and range of the functions.

(c) Some candidates still were unable to interpret the product of the functions. Some candidates did not realise that $f(x) - g(x) = 0$ represented the points of intersection of the functions.

Suggestions for improvement

(a) When teaching trigonometric functions, teachers should discuss the effects of parameters *a, p* and *q* on the basic trigonometric graph.

(b) As indicated in the previous report, teachers should explain the characteristics of each basic trigonometric function and how these characteristics change when the basic graph is transformed.

(c) Teachers should also explain that $f(x) - g(x) = 0$ represents the points of intersection of f and g, i.e. where $f(x) = g(x)$.

(d) They should also explain that the function is positive when it lies above the x-axis, zero when it lies on the x-axis and negative when it lies below the x-axis.

QUESTION 6: TRIGONOMETRY

Common errors and misconceptions

(a) Some candidates were able to write down the length of LN in Q6.1.1.

(b) In Q6.1.2 most candidates were unable to use the given information to determine KN, the length between the two cameras.

(c) Most candidates did not attempt Q6.2 as they could not apply a congruency axiom to determine the length of RT and then apply the cosine rule.

Suggestions for improvement

(a) Learners should be able to apply basic trigonometric ratios to determine the unknown side and angle of a right-angled triangle given the length of one of the sides and one of the acute angles.

(b) Teachers should expose learners to problems that require them to add or subtract two lengths in order to calculate the required length.

(c) Teachers should revise congruency axioms and expose learners to questions that integrate different topics.

QUESTION 7: EUCLIDEAN GEOMETRY

Common errors and misconceptions

(a) Most candidates were unable to recall the statement of the theorem in Q7.1.

(b) Candidates were unable to provide the reason in Q7.2.1.

(c) Most candidates struggled to apply Circle Geometry theorems when a combination of these theorems was required to answer a question.

Suggestions for improvement

(a) Learners should be encouraged to state theorems in full when they are engaging with questions on Euclidean Geometry.

(b) Teachers should give learners more exercises to practise the application of circle theorems.

QUESTION 8: EUCLIDEAN GEOMETRY

Common errors and misconceptions

(a) In Q8.2.2 most candidates managed to identify angles having a magnitude of 11° but struggled to give valid reasons for their statements.

(b) Most candidates did not attempt the rest of the questions.

Suggestions for improvement

(a) Teachers should revise theorems done in lower grades and show learners how to apply them as they move to Grades 11 and 12.

(b) Learners should practise using the 'acceptable reasons' in classwork so that they get familiar with them.

(c) Learners should be encouraged to indicate and add information on their diagrams as they unpack and solve riders.

QUESTION 9: EUCLIDEAN GEOMETRY

Common errors and misconceptions

(a) Most candidates were able to recall congruency axioms in Q9.2.1. This prevented them from determining the length of BE.

(b) Most candidates were unable to apply the proportionality theorems to calculate the lengths of AB, AT and FT.

Suggestion for improvement

Teachers should expose candidates to different ways of applying proportionality theorems even when the triangle has more than one pair of parallel sides given. They should be taught to view each triangle separately first, apply theorems and then look for connections.

QUESTION 10: CIRCLES, ANGLES AND ANGULAR MOVEMENT

Common errors and misconceptions

(a) Some candidates struggled with conversion from revolution per minute to radians per second in Q10.1.1.

(b) In Q10.1.2 most candidates used the correct formula but some made incorrect substitutions. Some calculated the two values of *h* correctly but did not indicate which of them is the correct length of PS.

(c) Some candidates were still unable to convert an angle from degree to radians.

(d) Most candidates were unable to use ratios to determine the radius of the smaller gear in Q10.2.4.

Suggestions for improvement

(a) Teachers should revise the conversion of angles and time to different units as it is key in rotation, angles and angular movement. Some formulae require angle in radians and some formulae require time in seconds.

(b) Learners should be exposed to high-order questions that require integration of different topics as was the case in Q10.2.4.

QUESTION 11: MENSURATION

Common errors and misconceptions

(a) There was confusion among many candidates. They were unable to derive the correct formulae for the calculation of surface area and volume of various objects.

(b) In Q11.1 some candidates copied the mid-ordinate formula incorrectly and some learners confused the value of "*a*" (the width) and the length of the rectangular portion.

(c) Most candidates could not determine the area that was not covered with grass.

(d) In Q11.2.1 most candidates did not subtract the area of the opening from the total area. Some candidates confused π as an angle in radian measure and not π as a number approximated to $\frac{22}{7}$.

(e) In Q11.2.2 candidates did not express volume of the container and cylindrical bottle in the same unit.

Suggestions for improvement

(a) Teachers need to encourage learners to use the information sheet and remind them to copy the formula correctly. They should also explain each of the variables in the formula.

(b) Learners should be exposed to all the prescribed basic shapes and appropriate formulae for them. Once learners have gained confidence with working with the basic shapes, then teachers should consider alterations to the basic shapes.

(c) Learners should be taught that when they compare objects in general, they should first express them in the same unit.

CHAPTER 3

TECHNICAL SCIENCES

The following report should be read in conjunction with the Technical Sciences question papers of the NSC November 2021 examinations.

3.1 PERFORMANCE TRENDS (2018–2021)

In 2021, 14 642 candidates sat for the Technical Sciences examination, 2 987 more candidates than in 2020.

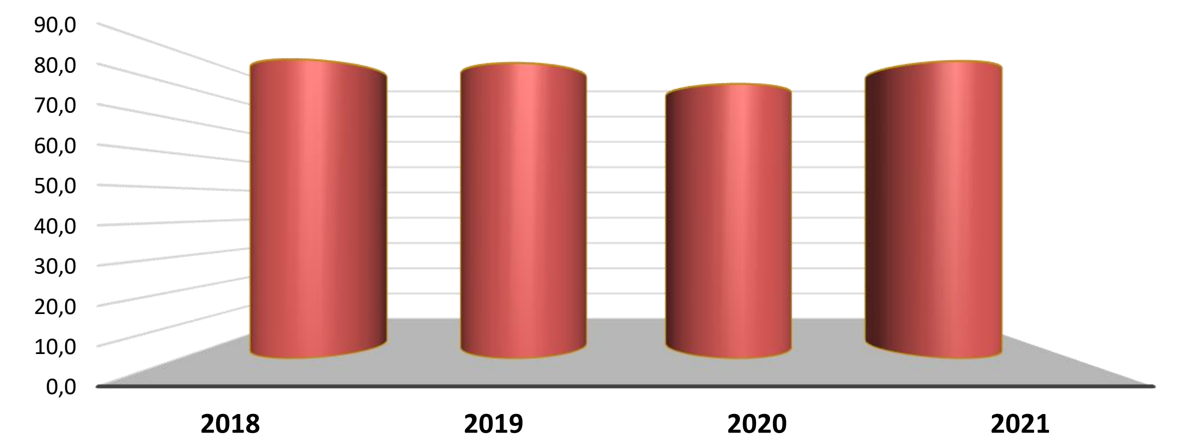
The performance of the candidates in 2021 shows a significant increase when compared to the performance in 2020. The pass percentage at 30% (Level 2) increased from 80,4% in 2020 to 87,1% in 2021.

It was very encouraging that 24,3% of candidates achieved over 50% this year in comparison with 18,2% of candidates doing so in 2020. The percentage of distinctions (80% and above) decreased marginally from 0,4% in 2020 to 0,3% in 2021.

Table 3.1.1 Overall achievement rates in Technical Sciences

Year	No. wrote	No. achieved at 30% and above	% achieved at 30% and above
2018	10 503	9 204	87,6
2019	10 862	9 401	86,5
2020	11 655	9 375	80,4
2021	14 642	12 758	87,1

Graph 3.1.1 Overall achievement rates in Technical Sciences (percentage)



	2018	2019	2020	2021
% achieved at 30% and above	87,6	86,5	80,4	87,1