

CHAPTER 10

MATHEMATICS

The following report should be read in conjunction with the Mathematics question papers for the NSC November 2021 examinations.

10.1 PERFORMANCE TRENDS (2017–2021)

The number of candidates who sat for the Mathematics examination in 2021 increased by 25 828 compared to that of 2020, i.e. 11% of the cohort.

The table below indicates variations in the pass rates over the past five years within a band of six percentage points at 30% (Level 2) and three percentage points at 40% (Level 3). However, there was a pleasing improvement in the pass rate this year.

Candidates who passed at 30% (Level 2) improved from 53,8% in 2020 to 57,6% in 2021. There was a corresponding improvement at 40% (Level 3) from 35,6% to 37,6%. Given the increase in the size of the cohort, the number of passes increased considerably by 23 651 at 30% (Level 2) and by 14 597 at 40% (Level 3).

The percentage of distinctions (over 80%; Level 7) declined marginally from 3,2% to 2,9%. Given the increased size of the cohort, this converts into an increase in the total number of distinctions from 7 466 in 2020 to 7 515 in 2021.

The results reflected above were despite the challenging circumstances brought about by the Covid-19 pandemic over the past two years which affected teaching and learning activities of the 2021 cohort. This appears to have been the result of constructive intervention strategies by teachers and subject advisors as well as schools and provincial education departments. The resourcefulness and diligence of the above-average candidates also contributed to the overall performance in the subject.

Table 10.1.1 Overall achievement rates in Mathematics

Year	No. wrote	No. achieved at 30% and above	% achieved at 30% and above	No. achieved at 40% and above	% achieved at 40% and above
2017	245 103	127 197	51,9	86 096	35,1
2018	233 858	135 638	58,0	86 874	37,1
2019	222 034	121 179	54,6	77 751	35,0
2020	233 315	125 526	53,8	82 964	35,6
2021	259 143	149 177	57,6	97 561	37,6

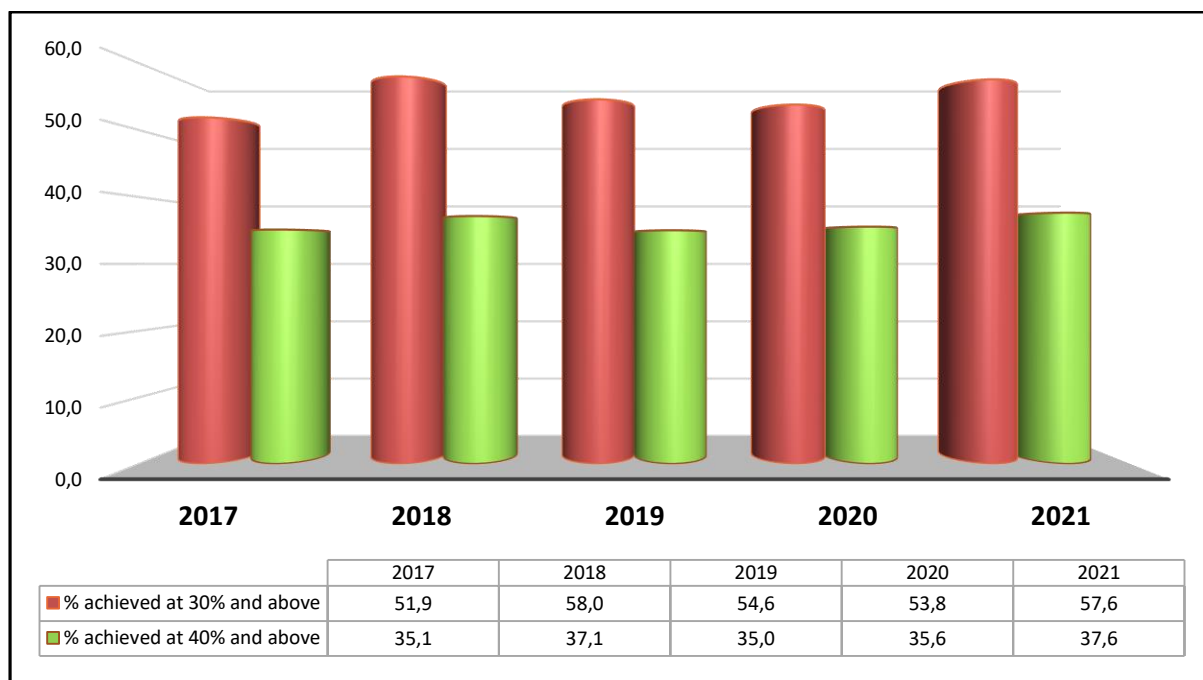
Performance in the 2021 examination revealed a slight improvement of candidates' understanding of basic concepts across some topics in the curriculum.

It appears that candidates are becoming over-reliant on past examination papers. While past examination papers may serve as a valuable resource for revision, the teaching and learning of basic concepts cannot be overlooked. It was pleasing to note that the candidates' answering of routine questions in Euclidean Geometry shows continuous improvement.

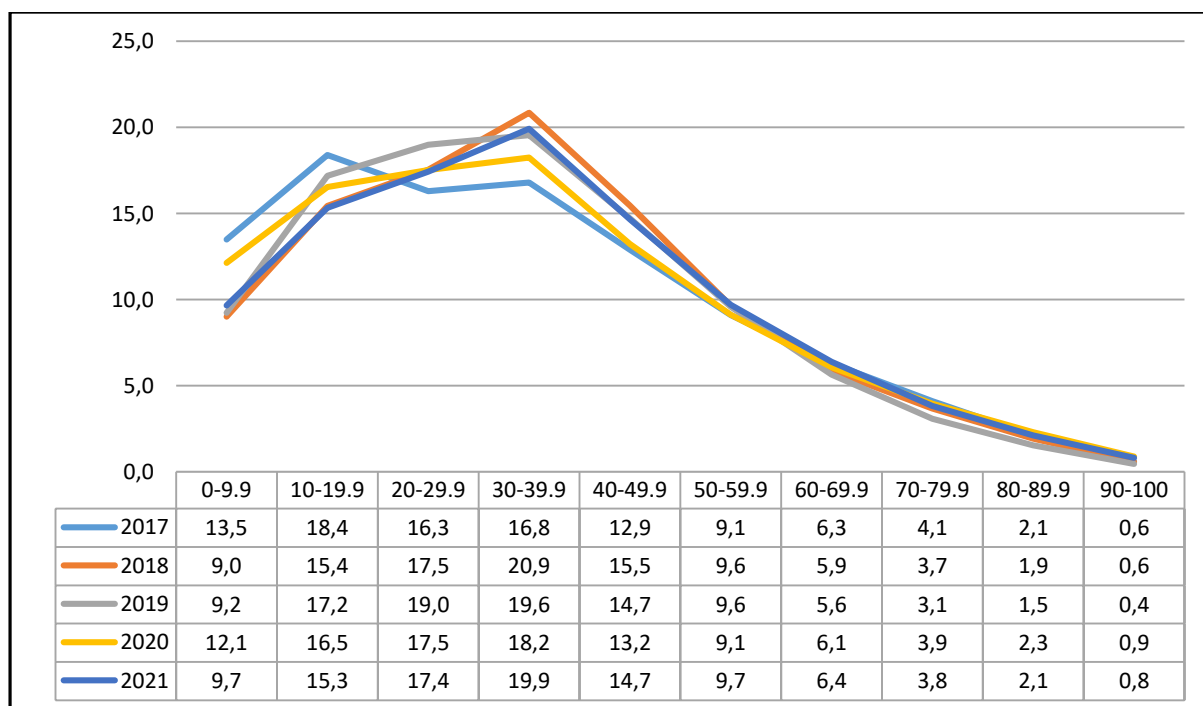
The CAPS states that the theory in Trigonometry is examinable. Teachers are reminded not to overlook this aspect when teaching solution of triangles and compound and double angles.

Performance will be further enhanced if attention is given to the following areas: strengthening the content knowledge in Trigonometry and learners' exposure to complex and problem-solving questions across all topics in the curriculum, starting in the earlier grades.

Graph 10.1.1 Overall achievement rates in Mathematics (percentage)



Graph 10.1.2 Performance distribution curves in Mathematics (percentage)



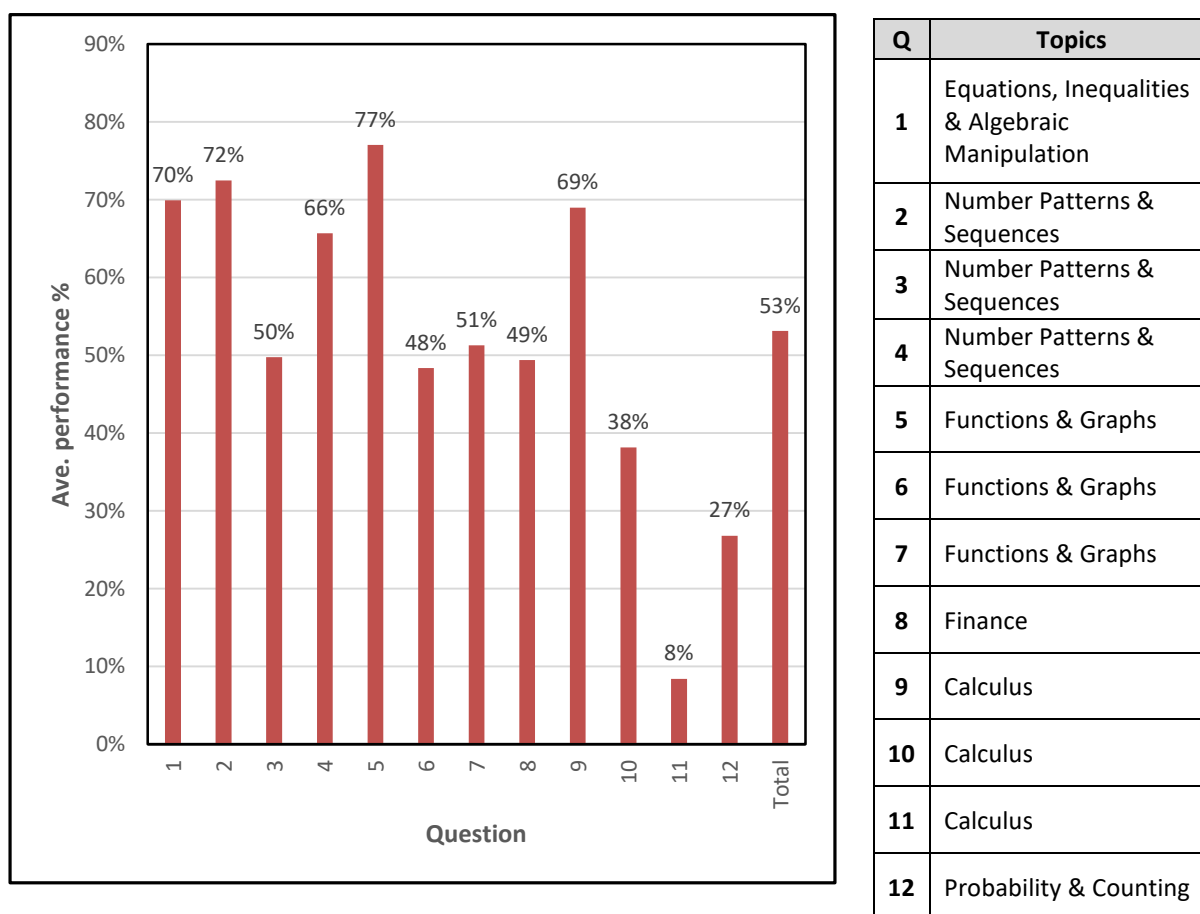
10.2 OVERVIEW OF CANDIDATES' PERFORMANCE IN PAPER 1

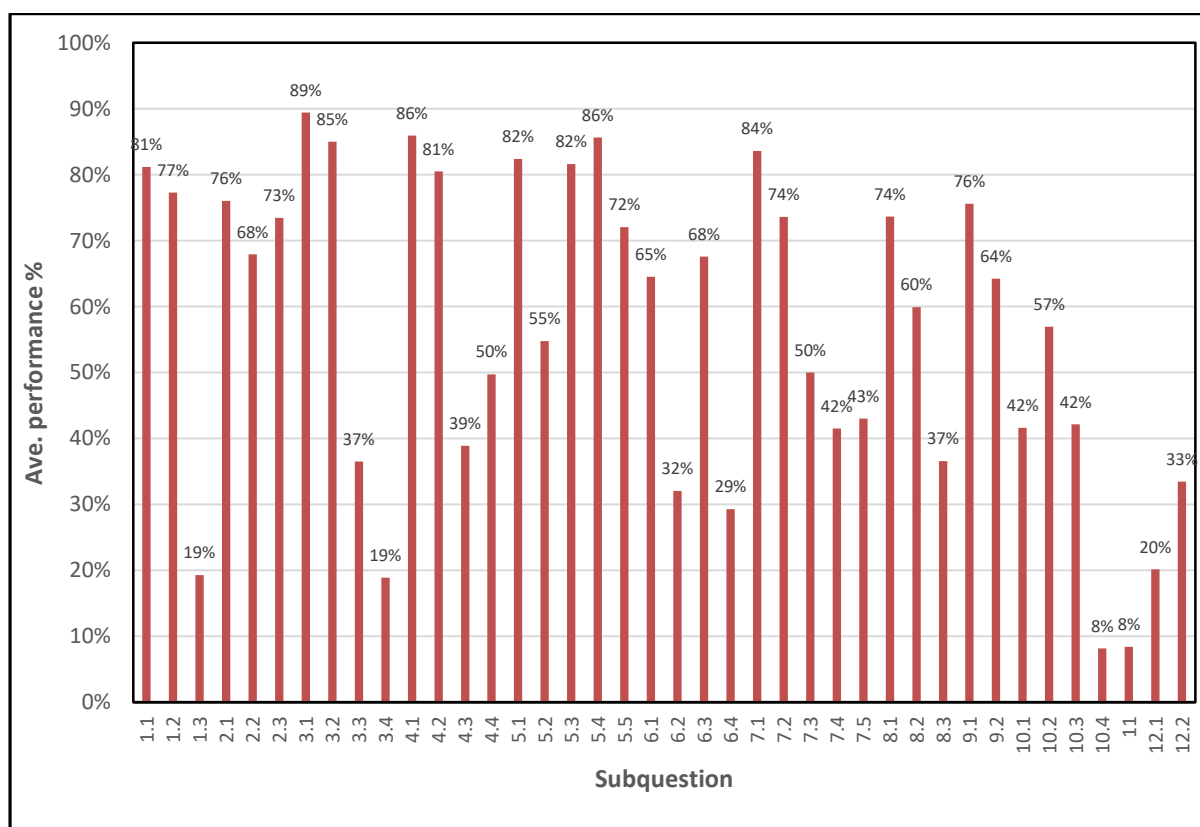
- (a) Many candidates were able to answer the knowledge and routine questions correctly and scored some marks in a majority of the questions. This suggests that the candidates were well prepared to deal with these questions in the paper.
- (b) The algebraic skills of the candidates are poor. Most candidates lacked fundamental and basic mathematical competencies which should have been acquired in the lower grades. This becomes an impediment to candidates answering complex questions correctly.
- (c) While calculations and performing well-known routine procedures form the basis of answering questions in a Mathematics paper, a deeper understanding of definitions and concepts cannot be overlooked. Candidates did not fare well in answering questions that assessed an understanding of concepts.

10.3 DIAGNOSTIC QUESTION ANALYSIS OF PAPER 1

The following graph is based on data from a random sample of candidates. While this graph might not accurately reflect national averages, it is useful in assessing the relative degrees of challenge of each question as experienced by candidates.

Graph 10.3.1 Average performance per question in Paper 1



Graph 10.3.2 Average performance per subquestion in Paper 1

10.4 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION IN PAPER 1

QUESTION 1: ALGEBRA

Common errors and misconceptions

- (a) Some candidates still factorised incorrectly in Q1.1.1.
- (b) Rounding off the answers to two decimal places is still a problem for some candidates. For example, in Q1.1.2 some candidates rounded to $-0,68$ instead of $-0,69$. Others simply rounded to $-0,6$ despite the question stating explicitly to TWO decimal places.

Candidates made the following errors when entering the values into the calculator:

- Omitting brackets around the -3 , i.e. $x = \frac{-3 \pm \sqrt{-3^2 - 4(2)(-3)}}{2(2)}$. This resulted in the following incorrect answers: $x = 0,22$ or $x = -1,72$.
- Creating the fraction for the part under the square root only, i.e. $x = -(-3) \pm \frac{\sqrt{(-3)^2 - 4(2)(-3)}}{2(2)}$. This led to the following incorrect answers: $x = 4,44$ or $x = 1,56$.

- (c) In answering Q1.1.3 many candidates treated the inequality as an equation. Their answer would read: $(x+1)(x+4) \leq 0$ followed by $x \leq -1$ or $x \leq -4$. These candidates

did not realise that the question dealt with the product of two numbers and that the product of two negative numbers does not yield a negative result. In addition, the difference in the solutions: $x \geq -4$ or $x \leq -1$ and $x \geq -4$ and $x \leq -1$ were not understood by a number of candidates.

Many candidates struggled to interpret the correct answer from the inequality.

$$\begin{array}{ll}
 x^2 + 5x + 4 \leq 0 & x^2 + 5x + 4 \leq 0 \\
 (x+1)(x+4) \leq 0 & \text{or} \quad (x+1)(x+4) \leq 0 \\
 \therefore x = -1 \text{ or } x = -4 & \therefore x = -1 \text{ or } x = -4 \\
 \therefore x \leq -4 \text{ or } x \geq -1 & \therefore -1 \leq x \leq -4
 \end{array}$$

Some candidates drew a sketch but were unable to use it to write down the required answer.

- (d) Most candidates had some idea that they had to square both sides of the equation in Q1.1.4. Few candidates were unable to square the binomial on the RHS correctly, for example, they wrote $x+28=4+x^2$ or $x+28=4-x^2$ instead of $x+28=4-4x+x^2$. Very few candidates checked if the solutions obtained were valid in the original equation and consequently failed to reject $x=8$ as a solution.
- (e) In Q1.2 some candidates made the following error when rewriting the linear equation in terms of one variable: $x=3-2y$. Other candidates overlooked the factor of y in the first term when substituting into the quadratic equation. They would write $2(2y-3)+7=(2y-3)^2+4y^2$ instead of $2(2y-3)y+7=(2y-3)^2+4y^2$. Some candidates used the quadratic formula to solve the equation $4y^2-6y+2=0$. However, they wrote their answer as $x=\frac{1}{2}$ or $x=1$ instead of $y=\frac{1}{2}$ or $y=1$.
- (f) Many candidates did not know how to answer Q1.3. Few candidates managed to arrive at $\Delta=-3n^2$, but could not explain why the roots were non-real. A fair number of candidates took arbitrary values for m , n and p and proved that n^2-4mp was negative. This was not acceptable.

Suggestions for improvement

- (a) Much of the work in this question is covered in Grade 11. It is therefore important for teachers to set revision tasks in these sections of work throughout the Grade 12 year.
- (b) More thorough teaching of factorisation in Grades 9 and 10 is needed. Emphasis should be placed on how to identify the type of factorisation that is applicable to the given expression. Encourage weaker learners to use the quadratic formula instead of factorising.
- (c) It is unacceptable for learners to write down the quadratic formula incorrectly. Therefore, they should be encouraged to copy the formula from the information sheet. Correct substitution, especially using brackets for negative values, should be emphasised in Grade 11. If this is done correctly, then learners should enter the values exactly as they have written it into their calculators to obtain the answers.

- (d) Teachers should not take for granted that learners know how to round off a number to the required number of places. Where necessary, this skill should be retaught in Grades 11 and 12.
- (e) Teachers should take some time, preferably in Grade 10, to focus on teaching learners how to represent inequalities (e.g. $-2 < x < 1$; $x < -2$ or $x > 1$) on a number line and also how to write an inequality from the illustration on a number line. This will benefit learners as they are required to write inequality solutions for a number of questions in both examination papers. Emphasise that correct notation is essential when writing down the solutions to inequalities.
- (f) Teachers should explain the difference between *and* and *or* in the context of inequalities. Learners cannot use these words interchangeably as they have different meanings.
- (g) When dealing with surd equations, learners should be reminded that they need to square both sides of the equation in order to maintain the balance. They should not square the radical parts of the equation only. Teachers must emphasise that implicit restrictions are placed on surd equations and that learners should continue to test whether their answers satisfy the original equation.
- (h) Teachers should emphasise the difference between non-real and undefined numbers as these are two different groups of numbers.

QUESTION 2: PATTERNS

Common errors and misconceptions

- (a) The question indicated that the given sequence was geometric. Despite this, when answering Q2.1, some candidates incorrectly assumed that it was arithmetic and calculated the value of x using a common difference between the terms:

$$T_2 - T_1 = T_3 - T_2 = d$$

$$90 - x = 90 - 81$$

$$x = 81$$

- (b) Q2.2 required candidates to calculate the sum of the first n terms of a geometric series. This is a well-known concept. However, many candidates found difficulty in answering the question because they had to show that $S_n = 1000(1 - 0,9^n)$. Some candidates used the T_n formula for a geometric sequence, whilst others used the sum formula for an arithmetic series despite the question indicating otherwise.
- (c) Candidates who assumed that the series was arithmetic, calculated the value of r to be -9 and subsequently used this value when calculating the sum to infinity. These candidates failed to realise that this value of r violated the condition for which a geometric series converges, namely $-1 < r < 1$. Other candidates used the incorrect value of $\frac{10}{9}$ for r .

Suggestions for improvement

- (a) At some stage it is advisable to give learners an exercise that contains a mixture of quadratic, arithmetic and geometric sequences and series. Learners should analyse the type of sequence they are working with and which formulae are applicable to it.

- (b) Teach learners how to identify whether the question requires them to calculate the value of the n^{th} term or the sum of the first n terms.
- (c) While covering this section, teachers should emphasise the difference between the *position* and the *value* of a term in a sequence. Learners must read the questions carefully so that they know what is required of them.
- (d) Remind learners that n cannot be a negative number, zero or a fraction. When solving for n , learners should arrive at a natural number solution. If this is not the case, then they should know that they have made a mistake in their working.
- (e) Make learners acutely aware of which formulae in the information sheet apply to which type of sequence. It is good practice for learners to use the information sheet in class so that they become familiar with it.
- (f) It is important to demonstrate, by way of example, the concept of a convergent geometric series, first by taking a value of $r > 1$ and then taking a value of $-1 < r < 1$. This should alert learners to the condition for which a geometric series will converge.

QUESTION 3: PATTERNS

Common errors and misconceptions

- (a) In answering Q3.3, many candidates incorrectly assumed that -121 was a term in the quadratic sequence instead of it being a term in the sequence of first differences. Consequently, they tried to solve the equation $-n^2 + 26n - 170 = -121$. This was viewed as a breakdown. A fair number of candidates created this equation:

$$n = \frac{-26 \pm \sqrt{(26)^2 - 4(-1)(-170)}}{2(-1)} = 0. \text{ These candidates had no clue that the value of } n$$

in a sequence cannot be 0.

- (b) The crux to answering Q3.4 was to compare the value of the maximum terms in the given sequence and the new sequence. Many candidates failed to link quadratic number patterns with the quadratic function. Hence, this question was not answered by a large majority of candidates.

Suggestions for improvement

- (a) Remind learners that n cannot be a negative number, zero or a fraction. When solving for n , learners should arrive at a natural number solution. If this is not the case, then they have made a mistake in their working.
- (b) When teaching quadratic number patterns, it is essential to show learners how the formulae: $T_1 = a + b + c$, the first term of the first differences $= 3a + b$ and the second difference $= 2a$, are deduced.
- (c) The sequence of first differences of a quadratic number pattern form an arithmetic pattern. This implies that an arithmetic sequence is embedded within a quadratic number pattern. Learners must read the question very carefully in order to establish which pattern the question is making reference to. Glossing over words in the question leads to learners making incorrect statements.

QUESTION 4: PATTERNS**Common errors and misconceptions**

- (a) A small minority of candidates used incorrect formulae in Q4.1 and Q4.2.
- (b) In Q4.3 a number of candidates successfully calculated the first three terms of the series but forgot to calculate the last term. However, the vast majority of the candidates did not write their answer as a sum of these terms. They wrote their answer as 5 ; 7 ; 9 ; ... ; 10 003 instead of $5 + 7 + 9 + \dots + 10\,003$. Candidates failed to realise that sigma notation is a compact form of a series of terms.
- (c) Many candidates failed to interpret the sigma notation correctly in Q4.4. They failed to see that some terms in the second expansion would cancel some terms in the first expansion. Further, candidates failed to realise that the question could have been solved as two separate sums.

Suggestions for improvement

- (a) Teachers need to clarify that the sigma notation is a short-hand notation of a series of terms. Give learners enough examples where they have to expand the sigma notation. Use simple ones to start with, probably containing only a few terms. Also give them examples that do not represent arithmetic and geometric series.
- (b) Learners should also be exposed to writing a series in sigma notation.

QUESTION 5: FUNCTIONS (HYPERBOLA)**Common errors and misconceptions**

- (a) In Q5.1, instead of the correct answer of $x = 3$ and $y = 2$, some candidates gave as the answer: $p = 3$ and $q = 2$, or $x \neq 3$ and $y \neq 2$. None of these were accepted as correct. Some candidates incorrectly wrote the equation of the vertical asymptote as $x = -3$.
- (b) Candidates still confuse the domain with the range and consequently gave the incorrect answer of $y = 2$. Many candidates gave their answer as $x \in R$. This was not accepted as it is incorrect.
- (c) Candidates were unable to correctly solve the equation $\frac{-1}{x-3} + 2 = 0$ on account of poor simplification skills. Hence, they could not calculate the x-intercept correctly.
- (d) Many candidates were able to sketch the hyperbola having the correct increasing shape. However, they failed to label the asymptotes and the intercepts with the axes on their sketch graphs. They were not awarded marks for the asymptotes and intercepts with the axes because their sketches were not drawn to scale.

Suggestions for improvement

- (a) Teachers should pay attention to the concepts and definitions when teaching functions.

- (b) Teachers should spend some time discussing that all points on the x-axis have a y-coordinate of 0 and all points on the y-axis have a x-coordinate of 0. The domain is always a set of x-values and the range is always a set of y-values.
- (c) When teaching the hyperbola, start with the 'basic graph' $y = \frac{a}{x}$ and develop the general hyperbola $y = \frac{a}{x+p} + q$. This will enable learners to understand the effect of the changes in the variables a , p and q on the graph, its asymptotes and axes of symmetry.

QUESTION 6: FUNCTIONS (EXPONENTIAL AND INVERSE)

Common errors and misconceptions

- (a) Many candidates were unable to solve the logarithmic equation correctly in Q6.1. Some incorrect answers were:

$$\begin{array}{ccccc} 2 = \log_4 k & \text{or} & 2 = \log_4 k & \text{or} & 2 = \log_4 k \\ \therefore k = 2^4 & & \therefore 2 = 4^k & & \therefore 4 = 2^k \end{array}$$

- (b) In Q6.2 many candidates failed to interpret the question correctly, i.e. to determine the values of x when the value of y lies from -1 to 2 . They did not realise that they had to determine an x -value when $y = -1$. Consequently, they were unable to state the correct interval in terms of x . A common incorrect answer was $0 \leq x \leq 16$.
- (c) Many candidates understood that they had to swop x and y in order to obtain the inverse of the function f in Q6.3. However, poor conversion from logarithmic form to exponential form resulted in an incorrect answer in y -form.
- (d) Candidates could not visualise the answer to Q6.4 because the sketch of the inverse of f was not given. Many candidates resorted to calculating the answer algebraically but their solutions were incorrect.

Suggestions for improvement

- (a) Teachers should spend some time discussing logarithms as a topic. The skill of changing from the exponential form to the logarithmic form and vice versa must be emphasised. This skill is required for determining the equation of the inverse of an exponential graph as well as solving for n in financial questions that observe an exponential pattern.
- (b) Teachers should discuss the meaning of mathematical statements: $x < 0$, $x > 0$, $y < 0$, $y > 0$, etc. and show where these regions are represented in the Cartesian plane.
- (c) Teachers should remind learners that the product of two numbers is negative when one of the numbers is positive and the other is negative. Similarly, the product of two numbers is positive when both numbers are negative or when both numbers are positive.
- (d) Basic interpretation of graphs should start in Grade 10. Learners should then be able to approach questions in Grades 11 and 12 with a little more confidence.

QUESTION 7: FUNCTIONS (PARABOLA AND STRAIGHT LINE)**Common errors and misconceptions**

- (a) While many candidates were able to determine the answers to Q7.1, they did not give their answers in coordinate form as required.
- (b) In Q7.2 many candidates failed to use the most direct method of calculating the x-coordinate of the turning point C, i.e. using the x-intercepts calculated in Q7.1. Instead they performed additional calculations to arrive at this answer.
- (c) When answering Q7.3 some candidates gave their answer in terms of x instead of y. These candidates confused the range with the domain. A number of candidates excluded the turning point in their answer. They gave the answer as $y > -25$ instead of $y \geq -25$. Some mistook G to be the turning point and gave the answer as $y \geq -24$.
- (d) In Q7.4.1 some candidates confused the angle of inclination with the gradient. They incorrectly calculated the gradient of AE as $m = \tan^{-1}(14,04^\circ) = 85,93$.
- (e) Many candidates incorrectly assumed that T was the midpoint of B and C when answering Q7.4.2. They were unable to make the link between the gradient of the tangent and the derivative of the function f . Of those candidates who used the derivative in their answer, some equated the derivative to 0 instead of equating it to the gradient of the tangent.
- (f) In Q7.5 many candidates were unable to determine the equation of the straight line passing through K correctly. The challenge in this instance was that they were unable to establish the gradient of the line correctly. Consequently, they were unable to determine the x-coordinate of R by solving a set of equations simultaneously. Some candidates were able to calculate the equation of the line passing through K correctly but then took R to be the x-intercept of this line.

Suggestions for improvement

- (a) Teachers should spend some time discussing the basic concepts of functions: all points on the x-axis have a y-coordinate of 0 and all points on the y-axis have a x-coordinate of 0. The domain is always a set of x-values and the range is always a set of y-values.
- (b) Teachers should integrate the findings of the gradient of a tangent to a cubic function to a parabola. They should ensure that learners understand that the gradient of the tangent through the turning point of a parabola is zero.

QUESTION 8: FINANCE**Common errors and misconceptions**

- (a) In Q8.1 some candidates used the straight-line depreciation formula instead of the reducing-balance depreciation formula.
- (b) It was evident in Q8.2 that the candidates were struggling with the application of logarithms in solving questions. In instances where candidates used n as the number of compounding periods, some of them had difficulty in interpreting the final answer.

$$116\,253,50 = 75000 \left(1 + \frac{0,068}{4} \right)^n, \text{ was followed by:}$$

$$n = 25,99$$

$$\therefore n = 26 \text{ years}$$

The calculation is correct but n represented the number of quarters and not years. Some candidates rounded off their answers too early. This resulted in an error in the answer. A few candidates swapped the values of A and P when substituting into the formula.

- (c) The most common error in Q8.3.1 was that candidates incorrectly selected the present value formula to answer this question. It would seem that candidates immediately use the present value formula to any question in which the purchase of a house is mentioned. Some candidates left out the '– 1' in the future value annuity formula.
- (d) In Q8.3.2(a), where candidates used the Pv formula to calculate the outstanding balance, they used the incorrect value of n . They used $n = 252$, the number of payments made, instead of $n = 48$, the number of payments outstanding. In the case where candidates used the alternate formula to calculate the outstanding balance, they only calculated the value of the payments made inclusive of interest. They omitted to subtract this amount from the value of the loan inclusive of interest.
- (e) Very few candidates had any idea how to respond to Q8.3.2(b). Many calculated the balance after 252 payments and subtracted this amount from the original loan amount. They failed to take into consideration the total amount repaid over the period.

Suggestions for improvement

- (a) Learners should be exposed to an exercise in which they select the correct formula to each question.
- (b) Teachers should explain the difference in meaning between the rate of interest and the amount of interest paid.
- (c) It is essential for learners to be able to accurately change from exponential form to logarithmic form. Teachers should teach this concept thoroughly.
- (d) Learners need deeper insight into the relevance of each of the formulae and under which circumstances each can be used. The variables in each formula must be explained. More practice in Financial Mathematics is necessary so that learners can distinguish among the different formulae.
- (e) Discuss the two ways of calculating the outstanding balance of a loan. The first is when the number of payments made is known and the second is when the number of payments outstanding is known.
- (f) Teachers should demonstrate all the steps required when using a calculator. Learners should be penalised in formal assessment tasks at school for rounding off early.

QUESTION 9: CALCULUS**Common errors and misconceptions**

- (a) In Q9.1 many candidates made the following notational errors:

$f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ or $\lim_{h \rightarrow 0} = \frac{f(x+h) - f(x)}{h}$ or $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$. They lost a mark for these errors.

Some candidates made the following mistakes when removing brackets:

$$2(x+h)^2 = (2x+2h)^2, \quad -(2x^2-3x) = 2x^2-3x, \quad 2(x+h)^2 = 2x^2+2xh+2h^2 \text{ or } f(x+h) = 2(x+h)^2.$$

In other instances, candidates did not use brackets in the numerator:

$f'(x) = \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 - 3x}{h}$. This led to a breakdown in the answer.

- (b) The common error in Q9.2.2 was that candidates were unable to convert the two terms to the differentiable form, ax^n , on account of the fractions.

Candidates wrote $-\frac{\sqrt[3]{x}}{2}$ as $-2(x)^{\frac{1}{3}}$ or $-2(x)^{\frac{2}{3}}$ or $-\frac{(x)^{\frac{2}{3}}}{2}$ instead of $-\frac{(x)^{\frac{1}{3}}}{2}$.

They also wrote $\left(\frac{1}{3x}\right)^2$ as $3x^{-2}$ or $9x^{-2}$ instead of $\frac{1}{9}x^{-2}$.

Suggestions for improvement

- (a) Emphasis should be placed on the use of the correct notation when determining the derivative, either when using first principles or the rules.
- (b) Teachers should explain the need for brackets when determining the derivative from first principles. This prevents the incorrect simplification that follows.
- (c) To apply the rules of differentiation, learners need a strong background in basic algebraic operations, e.g. factorisation, converting surds to exponential form and simplification of algebraic fractions. These skills should be revised before learners are expected to differentiate examples that contain surds or when algebraic manipulation is required.

QUESTION 10: CALCULUS**Common errors and misconceptions**

- (a) In determining a and b in Q10.1, candidates had to derive two linear equations and solve them simultaneously. Most candidates managed to substitute the coordinates of the turning point into the given expression and obtain the first equation. They were unable to derive the second equation because it required them to make use of the derivative. Some candidates took $a = -1$ and $b = 6$ as given and used these values in the given expression. They then calculated that the turning points of the function were $(0; 0)$ and $(4; 32)$. This is considered a circular argument and is not acceptable.

- (b) Some candidates experienced difficulty in factorising the expression in order to calculate the coordinates of A.
- (c) Many candidates failed to translate the words in Q10.3 into mathematical language. They were unable to link an increasing function to where the value of y increases when moving from left to right on the x -axis. Candidates had little idea that the change in concavity occurs at the point of inflection on the graph. Many did not calculate the x -coordinate of the point of inflection when answering Q10.3.2.
- (d) Many candidates did not realise that they had to translate the given graph by 1 unit to the right to solve the question. A number of them attempted to solve the question algebraically but were unsuccessful in doing so because the value of $h(x - 1)$ was not known.

Suggestions for improvement

- (a) The focus when teaching cubic functions should not only be on calculating the critical points but also on interpreting the critical points on the graph. For example, what does it mean when we know that the x -coordinate of a turning point on a graph is 4?
- (b) When teaching graphs of cubic functions, teachers should inform learners of both methods of determining the x -coordinate of the point of inflection: solving for x in $f''(x) = 0$ as well as determining the x -value midway between the two turning points.
- (c) Teachers should teach concavity in such a way that learners can visually identify where a graph is concave up or concave down. In this way, learners should deduce that the point of inflection is critical to establishing the concavity of a cubic graph.

QUESTION 11: CALCULUS

Common error and misconception

The vast majority of the candidates did not attempt this question because they were unable to derive the cost function from the given information.

Suggestions for improvement

- (a) Learners appear to be dependent on the formulae being given when solving optimisation problems. It is advisable that learners interrogate the optimum function even when it is given in a question. This should help their conceptual development.
- (b) Teachers should ensure that there is enough time for learners to understand the application of Calculus fully.
- (c) Reading for understanding should be ongoing if learners are to improve their responses to word problems.

QUESTION 12: PROBABILITY

Common errors and misconceptions

- (a) Many candidates could not provide a reason why events A and B were not mutually exclusive in Q12.1.1.

- (b) In Q12.2.1(a) many candidates overlooked the fact that the events A and B were independent. In addition, candidates were not familiar with the concepts 'only A' and 'only B'.
- (c) Many candidates could not visualise which region was represented by 'not A and not B'.
- (d) Some candidates were confused about which books were being referred to in Q12.2.2, despite the question explicitly stating 'these 12 books'.
- (e) Many candidates were able to calculate the options for the first and last place and how the three novels could be arranged together. However, they were unable to calculate how the novels together with the remaining books could be arranged in the 10 places between the first and last places.

Suggestions for improvement

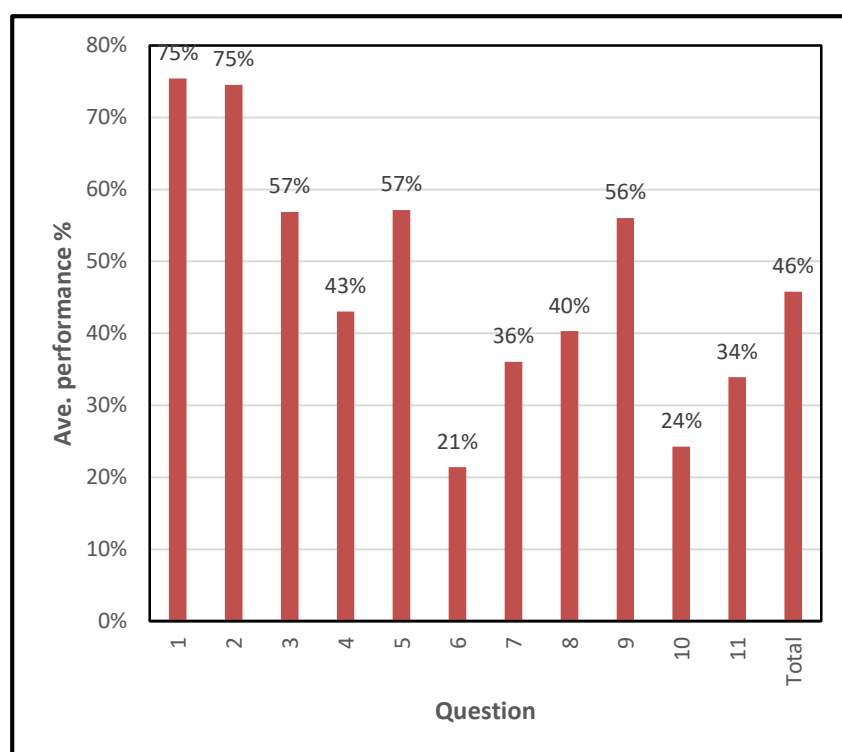
- (a) Teaching basic concepts cannot be overlooked. When learners understand the basic concepts well enough, then the more complex concepts are easier to grasp.
- (b) Use Venn diagrams to teach probability. It helps with the understanding of the different areas that make up the events, e.g. only A, only B, A and B, A or B, not A, not B, not A and not B and not A or not B.
- (c) Teach learners the Fundamental Counting Principle in such a way that they will be able to reason answers, instead of trying to remember rules.

10.5 OVERVIEW OF CANDIDATES' PERFORMANCE IN PAPER 2

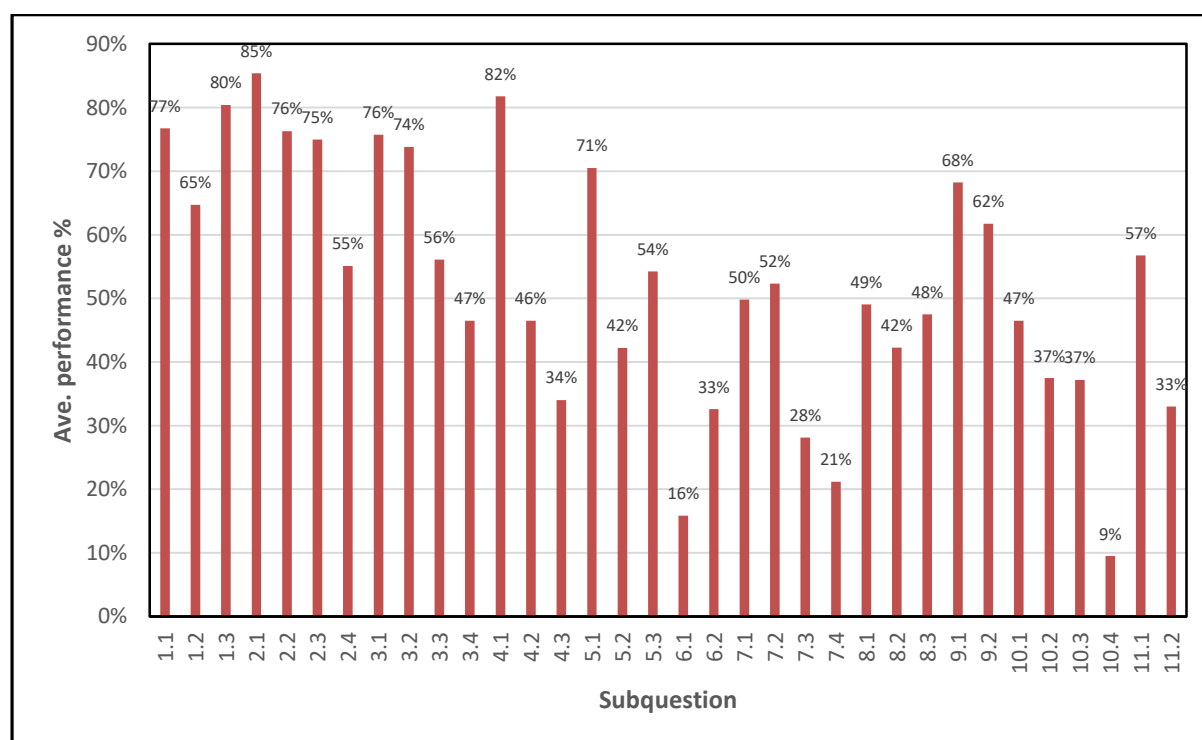
- (a) Individual performance in the paper varied from very poor to excellent.
- (b) Integration of topics is still a challenge to many candidates. Mathematics cannot be studied in compartments and it is expected that candidates will be able to apply knowledge from one section to another section of work.
- (c) It is evident that many of the errors made by candidates in answering the Trigonometry questions in this paper have their origins in a poor understanding of the basics and the foundational competencies taught in the earlier grades.
- (d) In general, candidates need to exercise caution with algebraic manipulation skills since overlooking certain basic principles or practices results in the unnecessary loss of marks. Although the calculator is an effective and necessary tool in Mathematics, learners appear to believe that the calculator provides the answer to all their problems. Some candidates need to realise that conceptual development and algebraic manipulation are often impeded because of the dependence on a calculator.

10.6 DIAGNOSTIC QUESTION ANALYSIS FOR PAPER 2

The following graph was based on data from a random sample of candidates. While this graph might not accurately reflect national averages, it is useful in assessing the relative degrees of challenge of each question as experienced by candidates.

Graph 10.6.1 Average performance per question in Paper 2

Q	Topic(s)
1	Data Handling
2	Data Handling
3	Analytical Geometry
4	Analytical Geometry
5	Trigonometry
6	Trigonometry
7	Trigonometry
8	Trigonometry
9	Euclidean Geometry
10	Euclidean Geometry
11	Euclidean Geometry

Graph 10.6.2 Average performance per subquestion in Paper 2

10.7 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION IN PAPER 2

QUESTION 1: DATA HANDLING

Common errors and misconceptions

- (a) Candidates were not careful when capturing the data into their calculators. This led to incorrect answers for the mean and standard deviation.
- (b) In Q1.1.1 some candidates calculated the median instead of the mean. Other candidates only added the top row of numbers as if the information given was a set of bivariate data. Some used the incorrect value of n in the denominator.
- (c) In Q1.1.2 some candidates calculated the mean or the correlation coefficient instead of the standard deviation. Some candidates selected the incorrect function on their calculators, i.e. they used s_x instead of σ_x .
- (d) When answering Q1.1.3 some candidates misread the question and calculated the number of days that fell between one standard deviation of the mean instead of one standard deviation above the mean. A few candidates only calculated the value of $\bar{x} + \sigma$ and did not calculate the number of days above this value.
- (e) Poor understanding of the question resulted in many candidates not answering Q1.2 correctly. Some candidates only calculated the average number of loaves of bread not sold per day instead of the total number of loaves not sold over the period.
- (f) In Q1.3.1 many candidates selected the correct box-and-whisker diagram that represented the data but they were unable to provide a reason for their choice. Candidates were unable to link the five-number summary to a box-and-whisker diagram.
- (g) In Q1.3.2 candidates lacked knowledge of skewness and were unable to describe the skewness of the box-and-whisker diagram that they had selected in Q1.3.1.

Suggestions for improvement

- (a) When teaching Statistics, the focus should not only be the calculations. Teachers should also pay attention to the meaning of the different concepts, e.g. mean, standard deviation, skewness, etc. The values obtained in the calculations should then become more meaningful for learners.
- (b) The understanding of statistical terminology is developed by using these terms frequently in the class. The use of diagrams when explaining the concepts of standard deviation and deviation intervals from the mean should help learners in understanding these concepts.
- (c) Practise calculator skills with learners. When calculating the standard deviation, the population standard deviation (σ_x) should be used and not the sample standard deviation (s_x). Learners are advised to become familiar with and use the same brand of calculator in the examinations.
- (d) Graphs are an integral part of Data Handling. Learners should be able to draw graphs, read off from graphs and interpret graphs.

QUESTION 2: DATA HANDLING (BIVARIATE DATA)**Common errors and misconceptions**

- (a) A few candidates were unable to plot all the points correctly in Q2.1. Some candidates joined the points or drew the regression line. The question did not require candidates to do either.
- (b) In Q2.2 some candidates did not calculate the values of a and b correctly. This was on account of entering incorrect values into the calculator. Some could not round off correctly to two decimal places. A few candidates swapped the values of a and b in the equation. Their equation was $y = 90,48x - 1,77$ instead of $y = 90,48 - 1,77x$. Some omitted the variable x in the equation and others did not start the equation with $y = \dots$
- (c) Some candidates substituted incorrectly in Q1.3, i.e. they substituted 5 for x instead of 38. Some candidates did not round off the answer to their calculation. These candidates failed to realise that it was not possible to sell 23,22 5-litre containers of milk.
- (d) Many candidates calculated the value of the correlation coefficient and then commented on the strength of the relationship between the price of milk and the number of containers sold. This was incorrect as it did not answer the question. Other candidates described the trend in the data. This was also incorrect. Candidates were expected to use the correlation coefficient to comment about the accuracy of the prediction.

Suggestions for improvement

- (a) Learners should be given multiple opportunities to practise calculator skills. Teachers should emphasise correct rounding procedures.
- (b) Teachers should explain each definition or concept in detail. Statistical language should be used in class so that learners become familiar with the terminology.
- (c) When determining the equation of the least squares regression line, it is advisable that learners write down the values of a and b and then write down the equation of the regression line. In this way, they can get the CA mark for the equation.
- (d) The teaching of Statistics goes beyond mere calculation of values. Learners should be able to use the values of their calculations to make predictions and comments about the data.

QUESTION 3: ANALYTICAL GEOMETRY**Common errors and misconceptions**

- (a) Many candidates failed to recognise that B, C and E were collinear points and hence, when answering Q3.1, failed to realise that $m_{BE} = m_{CE}$. Some substituted the coordinates of B into the gradient formula and ended up with an answer as an expression containing k . Some candidates still write the gradient formula incorrectly, despite it being given in the information sheet. Some candidates incorrectly used BE as the notation for the gradient of BE instead of m_{BE} .

- (b) In Q3.1.2 many candidates used $\tan^{-1}(81,87^\circ)$ to calculate the gradient of AB instead of $\tan 81,87^\circ$. This shows that candidates were confused between gradient and angle of inclination. Some candidates used the answers for k obtained in Q3.3.1 to calculate the gradient of AB. This was not accepted as the calculations for k were not done prior to answering Q3.1.2.
- (c) When answering Q3.2, some candidates calculated the y -intercept of BE correctly but failed to write down the equation of BE. Their answer was incomplete and they were not awarded the mark for the equation of BE.
- (d) In Q3.3.1 many candidates incorrectly assumed that C was the midpoint of BE. This information was not given and these candidates were not awarded any marks for their efforts.
- (e) Many candidates correctly calculated the gradient of AC as -2 but did not realise that this implied that $\hat{A}\hat{G}\hat{E}$ was obtuse. Although some candidates were able to calculate $\hat{A}\hat{G}\hat{E}$ and $\hat{A}\hat{F}\hat{G}$ correctly, they were unable to relate these angles to \hat{A} when answering Q3.3.2.
- (f) The point S was not shown on the sketch. Many candidates failed to attempt this question because they lacked the visual skills to correctly place point S in the first quadrant.
- (g) Many candidates did not substitute p for both x and y in the equation for BE. Consequently, they were unable to determine the coordinates of T as the equation contained two variables. Again, candidates lacked the visual skills to correctly place T in the first quadrant.
- (h) The centre of the circle was given. Many candidates were able to use this information to write down the LHS of the equation correctly in Q3.4.2(a). However, they did not realise that BE was the radius of the required circle.
- (i) Determining the equation of a tangent to a circle at the point of contact is a familiar question. However, many candidates lacked the visual skills to see that there was a circle passing through B and that they had to calculate the equation of the tangent passing through B.

Suggestions for improvement

- (a) If learners are not sure, they should consult the information sheet for the correct formula.
- (b) Substitution into the formula remains a problem. Learners should first write down the coordinates and then substitute them into the formula.
- (c) Teachers should request learners to label the coordinates as $(x_1; y_1)$ and $(x_2; y_2)$ on the diagram. This should prevent learners from making mistakes when substituting the coordinates into a formula. The order of substitution must be consistent, especially when using the gradient formula.
- (d) Emphasise to learners that it is not acceptable to make any assumptions, e.g. that a certain point is the midpoint of a line. Even if it looks as if the point is the midpoint, it

may not just be assumed and used. These need to be proved first before the results can be used in an answer.

- (e) Teachers should encourage learners to write down the values that they have already calculated (lengths, angles and gradients) on the diagram. This will assist learners when answering follow-up questions.
- (f) To answer questions in analytical geometry well, learners should master the properties of quadrilaterals and triangles. Constant revision of Analytical Geometry concepts taught in Grades 10 and 11 is essential, as much of the Grade 12 work revolves around these concepts.
- (g) The different topics in Mathematics should be integrated. Learners must be able to establish the connection between Euclidean Geometry and Analytical Geometry.
- (h) Learners have difficulty in visualising the figures and points not shown on a sketch. Teachers need to inculcate the skill of visualising and drawing the given information.

QUESTION 4: ANALYTICAL GEOMETRY

Common errors and misconceptions

- (a) Many candidates used the distance formula to calculate the radius. This was not necessary since the centre and the point A have the same x-coordinate, and all that was required was to subtract the y-coordinates of these two points.
- (b) In Q4.2.1 a number of candidates were unable to establish that BE and CD were both parallel to the x-axis and therefore these lines were perpendicular to CN, the radius of the circle. Consequently, they were unable to determine the coordinates of C.
- (c) Candidates were unable to make the link between the coordinates of C and the distance of 6 units in order to calculate the coordinates of D in Q4.2.2.
- (d) When answering Q4.2.3, many candidates had difficulty in identifying the height of $\triangle BCD$. A number of candidates used BD as the base but were unable to calculate the height of the triangle.
- (e) In answering Q4.3.1, some candidates could not recall the rule for reflecting a point about the line $y = x$. Many just swapped the signs without interchanging the x- and y-coordinates. Their coordinates of M were $(1 ; -3)$, which was incorrect.
- (f) Many candidates did not attempt Q4.3.2 because they could not place F on the diagram.

Suggestions for improvement

- (a) Teachers should encourage learners to analyse the diagram before attempting any questions. They must first write down any given information on the diagram and then make deductions from the given information.
- (b) Teachers need to revise the concept of perpendicular lines and gradients, particularly that the tangent is perpendicular to the radius at the point of contact. Teachers should also show learners why it is sufficient to subtract the x-coordinates to calculate the distance between two points in a horizontal plane and why it is sufficient to subtract the y-coordinates to calculate the distance between two points in a vertical plane.

- (c) Teachers should revise the work done in earlier grades. The properties of all the special quadrilaterals, e.g. the parallelogram, rhombus and square, should be taught thoroughly in earlier grades so that whenever that knowledge is needed, learners will be able to use it.
- (d) Learners should be reminded to refer to the information sheet for the relevant formula.
- (e) Teachers should show learners how to visualise and make rough drawings of all extra information given in Analytical Geometry questions.
- (f) Teachers should show learners different orientations of the base and the perpendicular height of a triangle. This should give learners more options when calculating the area of a triangle.
- (g) Teachers should ensure that they expose learners to assessments that integrate Analytical Geometry and Euclidean Geometry. Learners must also be exposed to higher-order questions in class and in school-based assessment tasks.

QUESTION 5: TRIGONOMETRY

Common errors and misconceptions

- (a) It was encouraging that many candidates had some knowledge to answer Q5.1. However, many candidates missed the minus sign in the reduction of $\tan(-x)$ and $\sin(360^\circ - x)$. When using the quotient identity to write $-\tan x$, some candidates used two negative signs, i.e. $-\tan x = \frac{-\sin x}{-\cos x}$ instead of $-\tan x = \frac{-\sin x}{\cos x}$. Some candidates omitted steps in their working, e.g. would simply state that $\frac{\sin 40^\circ}{\cos 50^\circ} = 1$ instead of writing $\frac{\sin 40^\circ}{\cos 50^\circ} = \frac{\sin 40^\circ}{\sin 40^\circ} = 1$. A small minority of candidates wrote $\frac{\sin}{\cos}$ instead of $\frac{\sin x}{\cos x}$, implying that they did not realise that a trigonometric ratio without an angle has no meaning.
- (b) In Q5.2 a number of candidates did not realise that they could replace $\sin^2 x$ in the numerator $-2\sin^2 x + \cos x + 1$ with the trigonometric identity $\sin^2 x = 1 - \cos^2 x$. Some of those who attempted to use this identity made mistakes in substitution, primarily because they did not use brackets as shown: $-2\sin^2 x = -2(1 - \cos^2 x)$. This resulted in incorrect simplification. Many candidates made mistakes in the reduction of $\cos(540^\circ - x)$. They could not work correctly with an angle greater than 360° .
Many candidates could not factorise $2\cos^2 x + \cos x - 1$ correctly. Some did not recognise it as a trinomial and incorrectly took out $\cos x$ as a common factor.
- (c) When answering Q5.3.1, many candidates opted to use $x^2 + y^2 = r^2$ instead of drawing the right-angled triangle. However, they made incorrect substitutions into this identity. Some candidates could not manipulate and simplify surds correctly. Some candidates gave the value of $\tan 36^\circ$ as $\frac{p}{\sqrt{1-p^2}}$ instead of $\frac{\sqrt{1-p^2}}{p}$.

- (d) The biggest challenge to answering Q5.3.2 was that candidates were unable to reduce $\cos 108^\circ$ to $\sin 36^\circ$. Some candidates correctly established that $\cos 108^\circ = -\sin 18^\circ$ and then incorrectly stated that $\sin 18^\circ = \frac{1}{2} \sin 36^\circ$. Some candidates wrote $\cos 108^\circ = \cos[3(36^\circ)]$ and then didn't know how to proceed from there or 'invented' their own triple-angle formulae.

Suggestions for improvement

- (a) Learners find it difficult to recall the Trigonometry taught in Grades 10 and 11. Revision of this work must be ongoing. It is better to revise small sections of work at a time than to give learners a comprehensive revision task.
- (b) Teachers should ensure that all learners are very confident in applying the definitions of the three trigonometric ratios in triangles.
- (c) Remind learners that the same simplification skills used in Algebra also apply to Trigonometry. Regular practice can remediate the poor algebraic and manipulation skills. Ensure that learners know how to correctly divide by a fraction, e.g.
- $$\frac{A}{\tan x} = \frac{A}{\frac{\sin x}{\cos x}} = A \times \frac{\cos x}{\sin x}.$$
- (d) Remind learners of the value of drawing a diagram when answering questions. This diagram could be a right-angled triangle or a Cartesian plane. A diagram should eliminate unnecessary identification errors. It is helpful to calculate the third angle in the right-angled triangle.
- (e) More emphasis should be placed on working with trigonometric ratios having angles greater than 360° and negative angles.

QUESTION 6: TRIGONOMETRY

Common errors and misconceptions

- (a) In Q6.1.1 many candidates were unable to derive the formula for $\cos(\alpha + \beta)$. Instead they merely copied the correct expansion from the information sheet. They were not awarded any marks for their efforts.
- (b) When answering Q6.1.2, many candidates did not pick up the clue from $2\cos 6x \cos 4x$ to write $\cos 10x$ as $\cos(6x + 4x)$. Many candidates wrote $\cos 10x$ as the double angle $\cos 2(5x)$, but this did not help in simplifying the given expression.
- (c) In answering Q6.2, some candidates wrote $2\sin 2x = 2\sin x \cos x$, i.e. they omitted the coefficient of 2 at the front of the expression. Almost all the candidates who managed to correctly arrive at $\sin x = 4\sin x \cos^2 x$ made the mistake of dividing both sides of the equation by $\sin x$. Some candidates overlooked the restriction that $\cos x < 0$.

Suggestions for improvement

- (a) The theory in Trigonometry cannot be overlooked. Teachers should ensure that this aspect is covered as stated in the CAPS.
- (b) When teaching compound angles, teachers need to stress that the trigonometric function cannot be distributed over two angles, e.g. that $\cos 10x = \cos(6x + 4x) \neq \cos 6x + \cos 4x$.
- (c) Teachers should explain to learners that dividing an equation by a trigonometric ratio results in the loss of possible solutions for equation. Moreover, a trigonometric ratio can be equal to zero for some values of the angle. By dividing an equation by a trigonometric ratio, the incorrect implication is that division by zero is permissible.
- (d) The key to solving trigonometric equations lies in understanding in which quadrants a trigonometric function is positive or negative. Also, it must be stated that $k \in \mathbb{Z}$ in the general solution as this qualifies the variable in the statement $+k \cdot 360^\circ$. It is advisable that learners be shown the graphical solution of trigonometric equations alongside the algebraic approach.

QUESTION 7: TRIGONOMETRY**Common errors and misconceptions**

- (a) In Q7.1 many candidates were unable to sketch the graph of $\cos(x - 60^\circ)$ correctly. Their graphs had incorrect turning points and x -intercepts. It would appear that candidates were using a calculator to generate the points on the graph but that these were not the critical points required for the sketch.
- (b) In Q7.2 some candidates multiplied 360° by 3 instead of dividing 360° by 3 when obtaining the period of $f(3x)$. In other instances, candidates incorrectly took the period of f to be 180° and, hence, incorrectly calculated the period of $f(3x)$ to be 60° .
- (c) Despite Q7.3 being familiar, candidates performed poorly in this question. They paid no attention to the mark allocation and attempted to solve the equation algebraically instead of using the graphs.
- (d) When answering Q7.4, many candidates gave the range of the original graph, g , instead of the transformed graph, k .

Suggestions for improvement

- (a) When teaching the drawing of trigonometric graphs, it is strongly recommended that the approach should be to ensure that learners know the basic graphs, the 'mother graphs' very well. Thereafter, teachers should explain how to draw the required graph by applying knowledge of transformations.
- (b) Although these concepts are discussed in Grade 10, it is necessary for learners to be constantly reminded of the meaning of concepts like period, domain, amplitude and range.
- (c) When teaching trigonometric functions to emphasise the meaning and effect of each of the parameters a , k , p and q in the equation $y = a \sin(kx + p) + q$, for example.

- (d) When discussing the transformed graphs, teachers should pay attention to how the characteristics of the original graph change and how the critical points of the original graph shift.
- (e) Learners should be told that the period of a trigonometric function is the length of a function's cycle. Since this value is a length, it is a single number and not an interval of values.

QUESTION 8: TRIGONOMETRY

Common errors and misconceptions

- (a) In Q8.1 some candidates incorrectly assumed that TQS was a right-angled triangle and used the definitions of trigonometric ratios instead of the sine rule to prove that $QS = 5 \tan x$.
- (b) Many candidates did not attempt Q8.2 because they had no idea where to start the answer. Those who correctly chose to use the cosine rule were unable to simplify their expressions to the final answer. Some candidates stated that $(5 \tan x)^2 = 10 \sin^2 x$ in their answer. This was incorrect.
- (c) A number of candidates could not link QT with the calculation of the area of ΔTQR . Some candidates incorrectly attempted to calculate the area of ΔTQR by using the formula: area of triangle = $\frac{1}{2}$ base \times height, where QR was the base and QT was the height. These candidates did not realise that QT was not perpendicular to QR.

Suggestions for improvement

- (a) A careful analysis of the information provided will give learners some idea of the concepts required in solving a triangle.
- (b) Teachers need to develop strategies to be used when solving right-angled triangles and triangles that are not right-angled. Teach learners the conditions that determine which rule should be used to solve the question.
- (c) It might be a good idea to give learners an exercise in which they identify which rule is to be used to solve the question. The learners must also give a reason why they think that the rule that they have selected applies to the question.
- (d) Learners should be encouraged to highlight the different triangles using different colours.
- (e) Initially, expose learners to numeric questions on solving 3D problems. This makes it easier for them to develop strategies on how to solve such questions. Once learners have gained confidence with numeric type questions, they should then be exposed to non-numeric and higher-order questions.

QUESTION 9: EUCLIDEAN GEOMETRY**Common errors and misconceptions**

- (a) Many candidates were unable to state the correct reason in Q9.1. Some stated that the chords from the same point were equal, whilst others just wrote down tangents. Neither of these were accepted as correct.
- (b) While candidates were able to write down the correct values of the answer in Q9.2.1, they were unable to state the correct reason. The reason 'isosceles triangle' was not accepted as correct. The expected reason was 'angles opposite equal sides' as stated in the *Examination Guidelines*.
- (c) In Q9.2.2 many candidates were unable to make correct relationships between the angles. Some candidates regarded \hat{S}_2 as an exterior angle of the cyclic quadrilateral instead of $\hat{S}_2 + \hat{S}_3$ being the exterior angle. Some candidates incorrectly stated that $\hat{Q} = \hat{S}_1$, i.e. that the opposite angles of the cyclic quadrilateral are equal. Some candidates refer to RSW as \hat{S}_{2+3} instead of $\hat{S}_2 + \hat{S}_3$. The notation \hat{S}_{2+3} is not accepted. Some candidates just gave the final answer as $\hat{S}_3 = 42^\circ$, without showing any steps or giving any reasons.

Suggestions for improvement

- (a) The key to answering Euclidean Geometry successfully is to be fully conversant with the terminology in this section. To this end, teachers should explain the meaning of *chord*, *tangent*, *cyclic quadrilateral*, etc. so that learners will be able to use them correctly.
- (b) Teachers must cover the basic work thoroughly. An explanation of the theorem should be accompanied by showing the relationship in a diagram.
- (c) Teachers are encouraged to use the Acceptable Reasons in the *Examination Guidelines* when teaching. This should start from as early as Grade 8.
- (d) Learners should be encouraged to scrutinise the given information and the diagram for clues about which theorems could be used in answering the question.

QUESTION 10: EUCLIDEAN GEOMETRY**Common errors and misconceptions**

- (a) In Q10.1 many candidates did not provide a correct or complete reason for their statements. They failed to mention that CM was the line from the centre. Some mistook AE for a tangent. Some candidates gave the incorrect reason: line from centre bisects the chord.
- (b) In Q10.2 some candidates confused the relationship between \hat{M}_1 and \hat{A}_1 . They stated that $\hat{M}_1 = \hat{A}_1$, i.e. that the angles in the same segment were equal. The correct relationship was that $\hat{M}_1 = 2\hat{A}_1$ because the angle at the centre is twice the angle at

the circle. Some candidates stated that $\hat{C} = \hat{E}$ and supported this with the reason that these were angles in the same segment. This was incorrect as these angles were not subtended by the same chord.

- (c) Many candidates provided the incorrect reason when answering Q10.3. They stated tan-chord theorem instead of converse: tan-chord theorem. Another concern was that candidates would make claims, e.g. AC is a diameter or $\hat{A}_2 = \hat{C} = x$, without providing a reason or calculation to justify their claim. It was not acceptable to state that $\hat{A} = 90^\circ$ as there were two right angles at the vertex A.
- (d) Only a few candidates were able to identify that the Midpoint Theorem has to be used in this question. Many candidates attempted to use proportionality and/or similar triangles but were unsuccessful in obtaining the correct answer.

Suggestions for improvement

- (a) Teachers should develop the skill in learners to analyse the question and the diagram for clues on which theorems are required to answer the questions correctly.
- (b) Learners should be forced to use acceptable reasons in Euclidean Geometry. Teachers should explain the difference between a theorem and its converse. They should also explain the conditions for which theorems are applicable and when the converse will apply.
- (c) Learners should be discouraged from writing correct statements that are not related to the solution. No marks are awarded for statements that do not lead to solving the problem.
- (d) Learners need to be told that success in answering Euclidean Geometry comes from regular practice, starting off with the easy and progressing to the difficult.

QUESTION 11: EUCLIDEAN GEOMETRY

Common errors and misconceptions

- (a) Some candidates did not show or state the construction in proving the theorem in Q11.1. No marks were awarded in this case. Other candidates made constructions that were not related to the proof. Some candidates stated that $\hat{F}_1 + \hat{F}_2 = 90^\circ$ but failed to label the angles as such in the diagram. Again, no marks were awarded for this statement.
- (b) In Q11.2.1(a) some candidates named the angles incorrectly, e.g. stating that $\hat{M} = \hat{S}$ instead of $\hat{M}_2 + \hat{M}_3 = \hat{S}_1$. Some candidates omitted the parallel lines when they used the reason that corresponding angles were equal.
- (c) Q11.2.1(b) was not answered by many candidates as they were unable to identify the correct relationship between the angles for KLMN to be a cyclic quadrilateral. Of those who were able to identify the correct relationship, some gave the incorrect reason. They stated that the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle instead of the exterior angle of a quadrilateral is equal to the interior opposite angle. There was confusion between the theorem and its converse.

- (d) In Q11.2.2 some candidates assumed that there will always be one common angle between the two triangles, and therefore incorrectly assumed that the angles formed at the common vertex K were equal. Many candidates knew which angles they were supposed to prove equal, but couldn't give the reasons why the angles were equal.
- (e) When answering Q11.2.3, some candidates wrote the correct statement, namely that $\frac{LK}{KS} = \frac{KN}{SM}$, but gave as reason the proportionality theorem instead of similar triangles. Some candidates incorrectly identified the corresponding sides of the similar triangles. Many candidates rewrote $3KN = 4SM$ as $\frac{KN}{SM} = \frac{3}{4}$, instead of $\frac{KN}{SM} = \frac{4}{3}$.
- (f) Many candidates failed to answer Q11.2.4. Some candidates provided as the reason the proportionality theorem but failed to mention the parallel lines in the reason. Some candidates applied the proportionality theorem to $\triangle PSK$ instead of $\triangle LMN$.

Suggestions for improvement

- (a) Learners should be taught that a construction is required in order to prove a theorem. If the construction is not shown, then the proof is regarded as a breakdown and they get no marks. Teachers should reinforce theory in short tests and assignments.
- (b) More time needs to be spent on the teaching of Euclidean Geometry in all grades. More practice on Grade 11 and 12 Euclidean Geometry will help learners to learn theorems and diagram analysis. They should carefully read the given information without making any assumptions. This work covered in class must include different activities and all levels of the taxonomy.
- (c) Teachers should require learners to make use of the diagrams in the Answer Book to indicate angles and sides that are equal and record information that has been calculated.
- (d) Learners need to be made aware that writing correct but irrelevant statements will not earn them any marks in an examination.
- (e) Learners should be taught that all statements must be accompanied by reasons. It is essential that the parallel lines be mentioned when stating that corresponding angles are equal, alternate angles are equal, the sum of the co-interior angles is 180° or when stating the proportional intercept theorem.
- (f) Learners need to be exposed to questions in Euclidean Geometry that include the theorems and the converses. When proving that a quadrilateral is cyclic, no circle terminology may be used when referring to the quadrilateral.