

# Gr 10-11

# Wiskunde

VRAESTEL 1

# INHOUD VERANDERINGS

## Statistiek uitgehaal

Gr 11:

### Scenario 1:

Gr 12 2021 – geen statistiek nie.

Geen bekommernis.

### Scenario 2:

Gr 12 2021 – wel statistiek.

- jaag en handel gr11 statistiek in 2020 af.

- Maak plek vir 2 weke se ekstra statistiek in gr 12.

Gr 10:

**Baie min werk, maklik om in gr 11 / 12 in te haal**

# VRAESTELLE VERDELING

	Gr 10	Gr 11
<b>ANALITIES</b>	20	35
<b>TRIGONOMETRIE</b>	45	60
<b>EUKLIDIES</b>	35	55
	100	150

# ANALITIES

2de  
kwartaal

Gr 10

**Gradient van 'n lyn:**

$$m_{AB} = \frac{y_A - y_B}{x_A - x_B}$$

**Lengte van 'n lynstuk:**

$$AB = \sqrt{(y_A - y_B)^2 + (x_A - x_B)^2}$$

**Koordinaat van middelpunt:**

$$\text{Midpt } AB = \left( \frac{x_A + x_B}{2}; \frac{y_A + y_B}{2} \right)$$

**Ewewydige lyne:**

$$m_1 = m_2$$

**Loodregte lyne:**

$$m_1 \times m_2 = -1$$

# ANALITIES

2de  
kwartaal

Gr 11

**Afstandsformule:**  $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

**Middelpuntsformule:**  $= \left( \frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right)$

**Gradient:**  $= \frac{y_2 - y_1}{x_2 - x_1}$

**Vergelyking van lyn:**  $\frac{y - y_2}{x - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$

**Inklinasiehoek:**  $m = \tan \theta$

# TRIGONOMETRIE

Gr 10

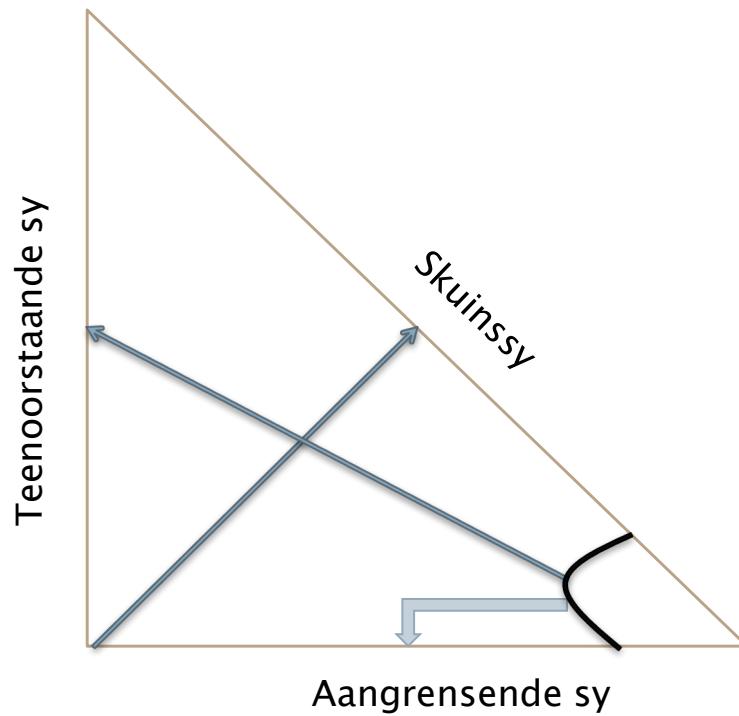
1ste EN  
3de  
kwartaal

1. Verstaan die begrip en rede van trig. Driehoek met verhoudings.
2. Doen herleidings en manipuleer trigonometriese uitdrukkings
3. Los die hoek op van 'n trigonometriese vergelyking
4. Teken die grafieke van trigonometriese uitdrukkings
5. Los probleme op.

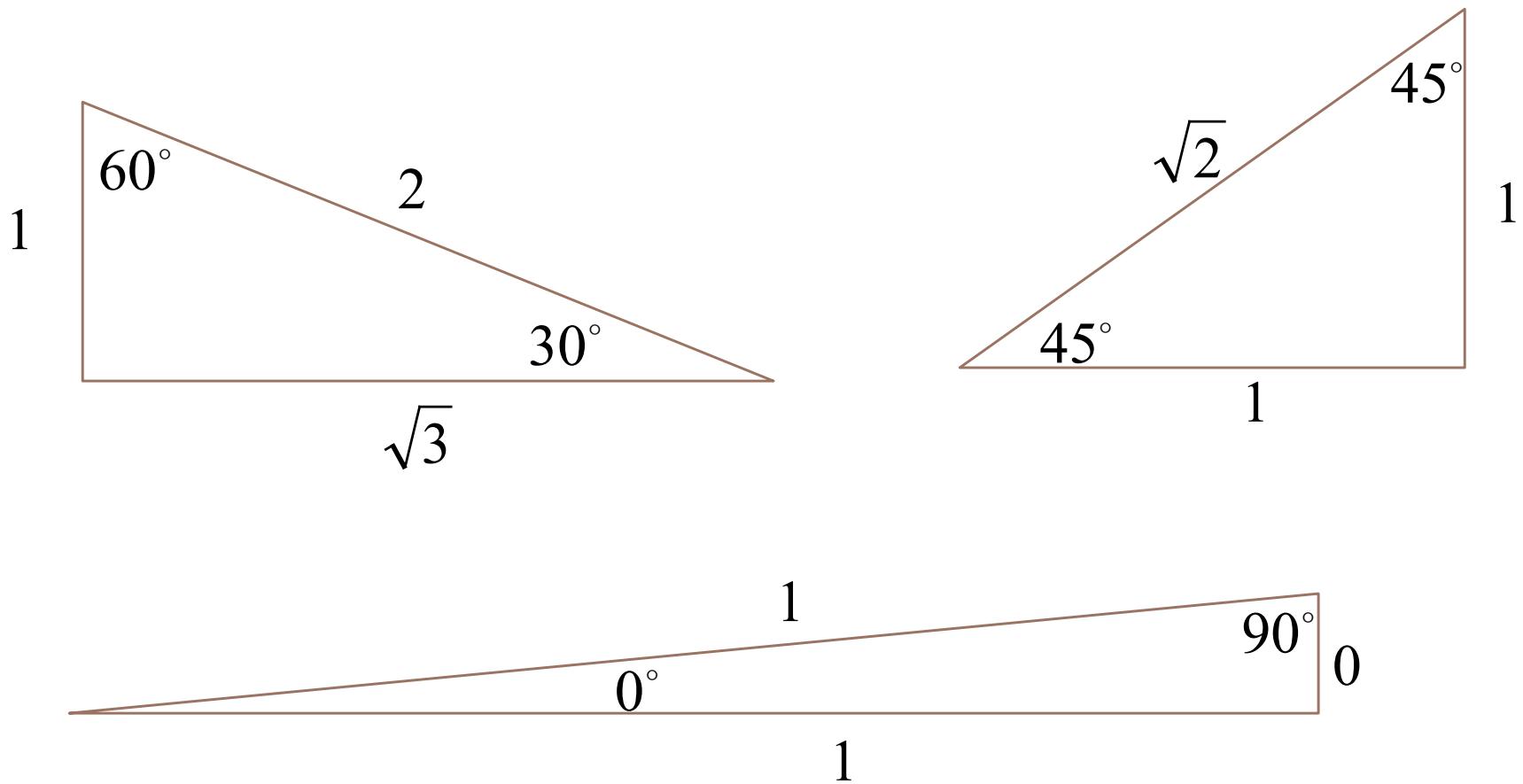
# TRIGONOMETRIE

Gr 10

1ste EN  
3de  
kwartaal



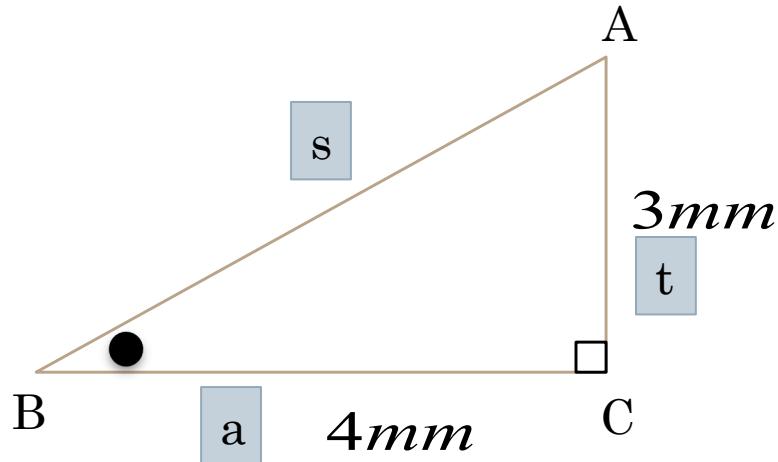
# Spesiale hoeke (Sonder sakrekenaar)



# Trigonometrie

## Reghoekige driehoek

Bereken die grootte van hoek B in die volgende driehoek:



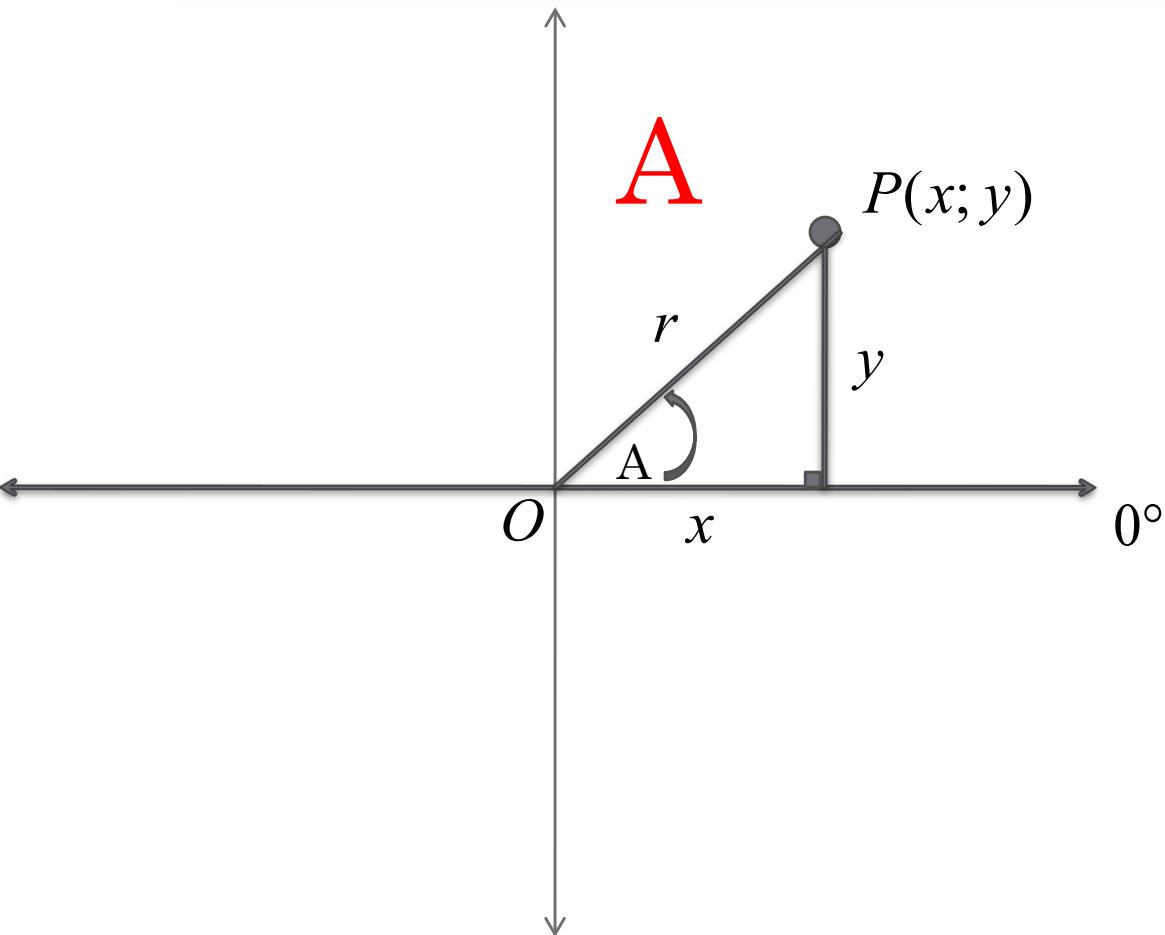
$$\tan \hat{B} = \frac{t}{a}$$

$$\tan \hat{B} = \frac{3\text{mm}}{4\text{mm}}$$

$$\begin{aligned}\tan \hat{B} &= 0,75 \\ \hat{B} &= 36,87^\circ\end{aligned}$$

# Trigonometrie

Definisies van die sinus, kosinus en tangens van hoeke van  
enige grootte



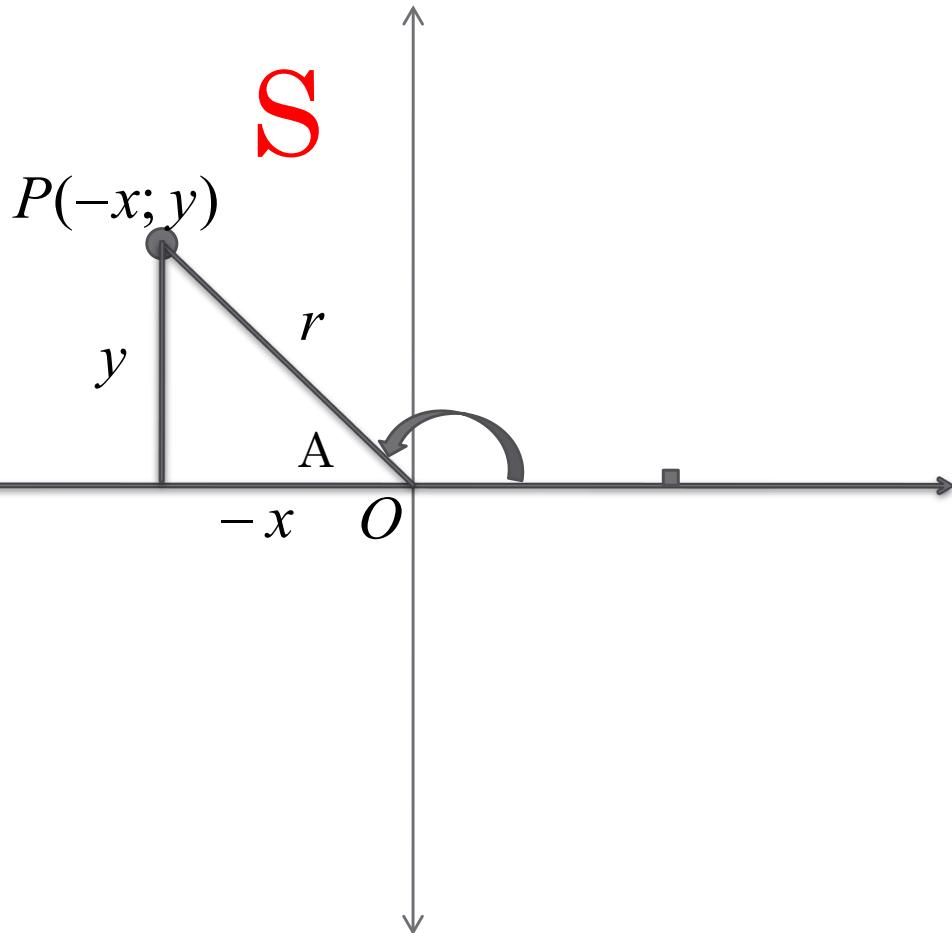
$$\sin \hat{A} = \frac{y}{r}$$

$$\cos \hat{A} = \frac{x}{r}$$

$$\tan \hat{A} = \frac{y}{x}$$

# Trigonometrie

Definisies van die sinus, kosinus en tangens van hoeke van  
enige grootte



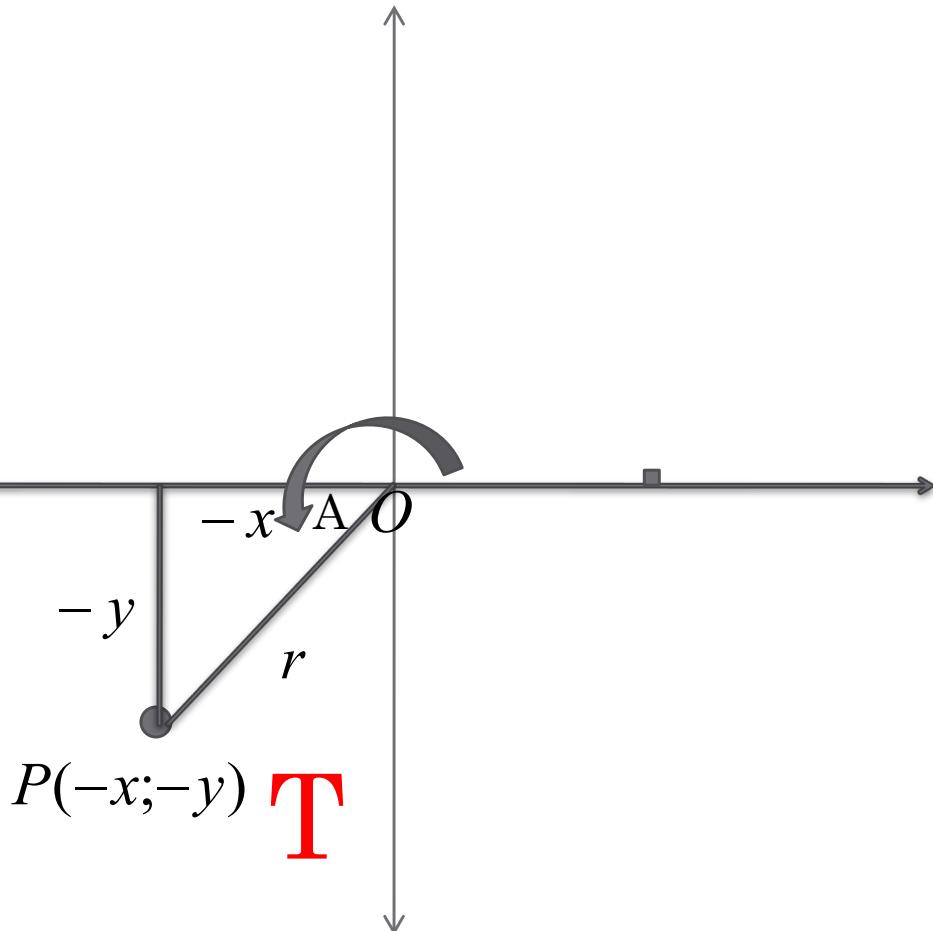
$$\sin \hat{A} = \frac{y}{r}$$

$$\cos \hat{A} = \frac{-x}{r}$$

$$\tan \hat{A} = \frac{y}{-x}$$

# Trigonometrie

Definisies van die sinus, kosinus en tangens van hoeke van  
enige grootte



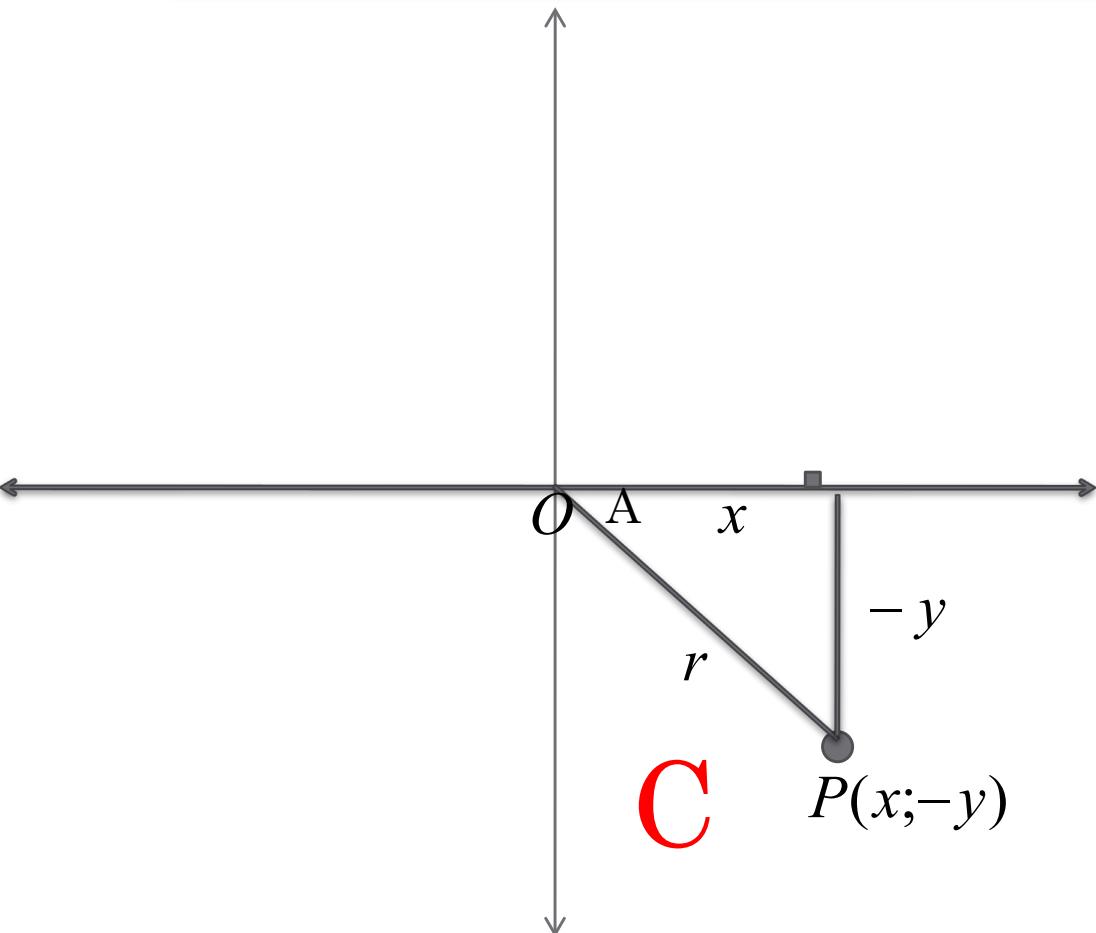
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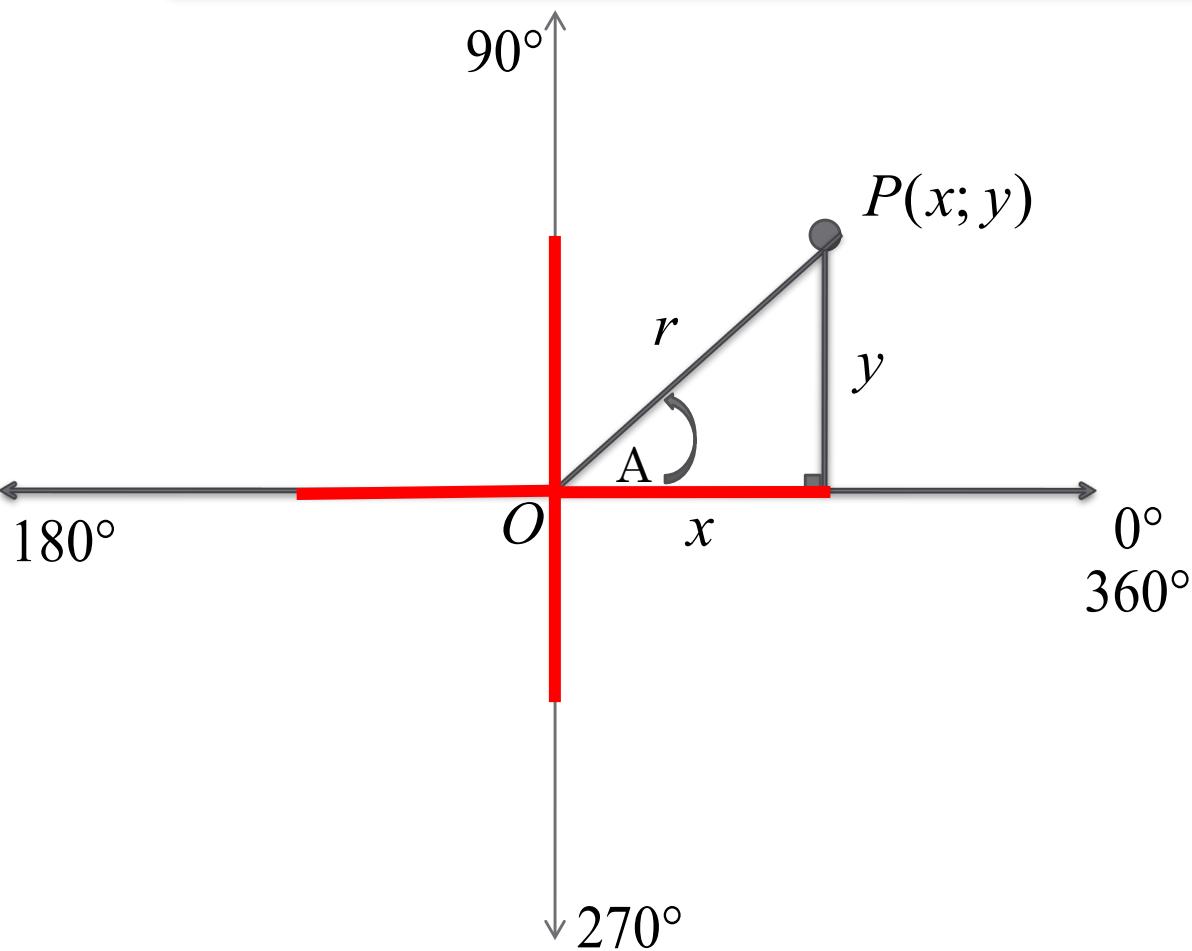
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# Trigonometrie

Definisies van die sinus, kosinus en tangens van hoeke van  
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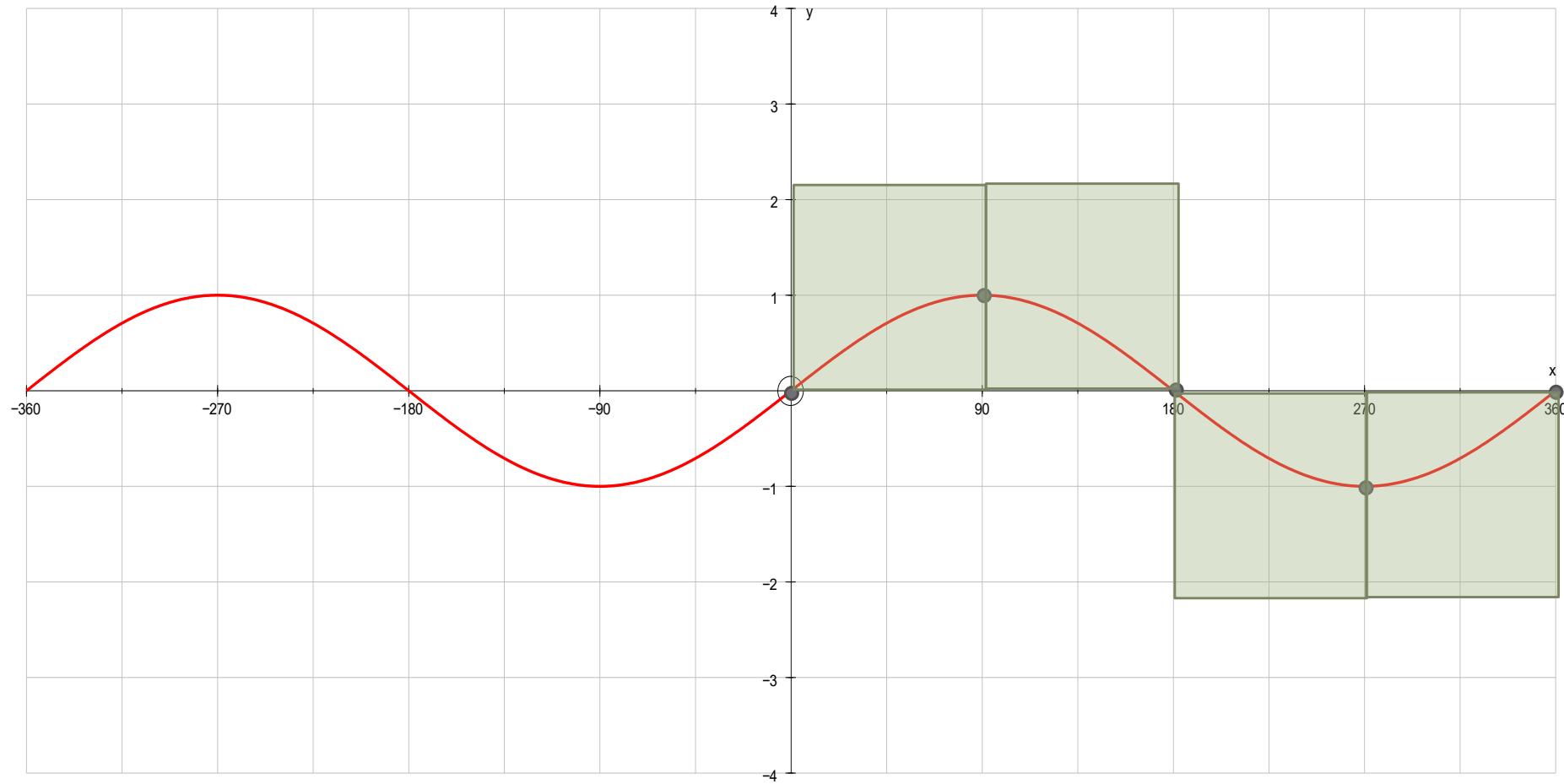


$$\sin \hat{A} = \frac{-y}{r}$$

$$\cos \hat{A} = \frac{x}{r}$$

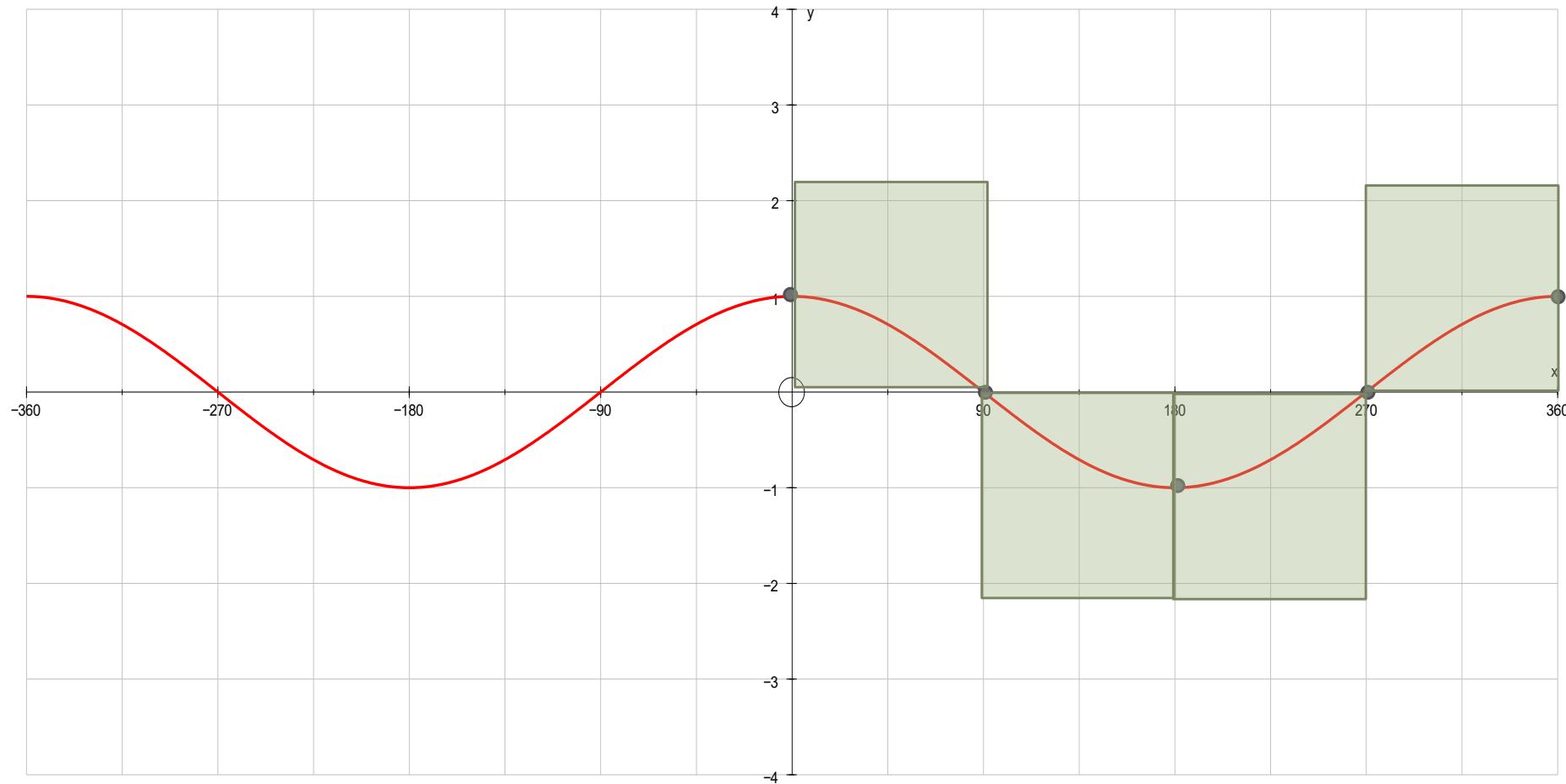
$$\tan \hat{A} = \frac{-y}{x}$$

# Trigonometrie



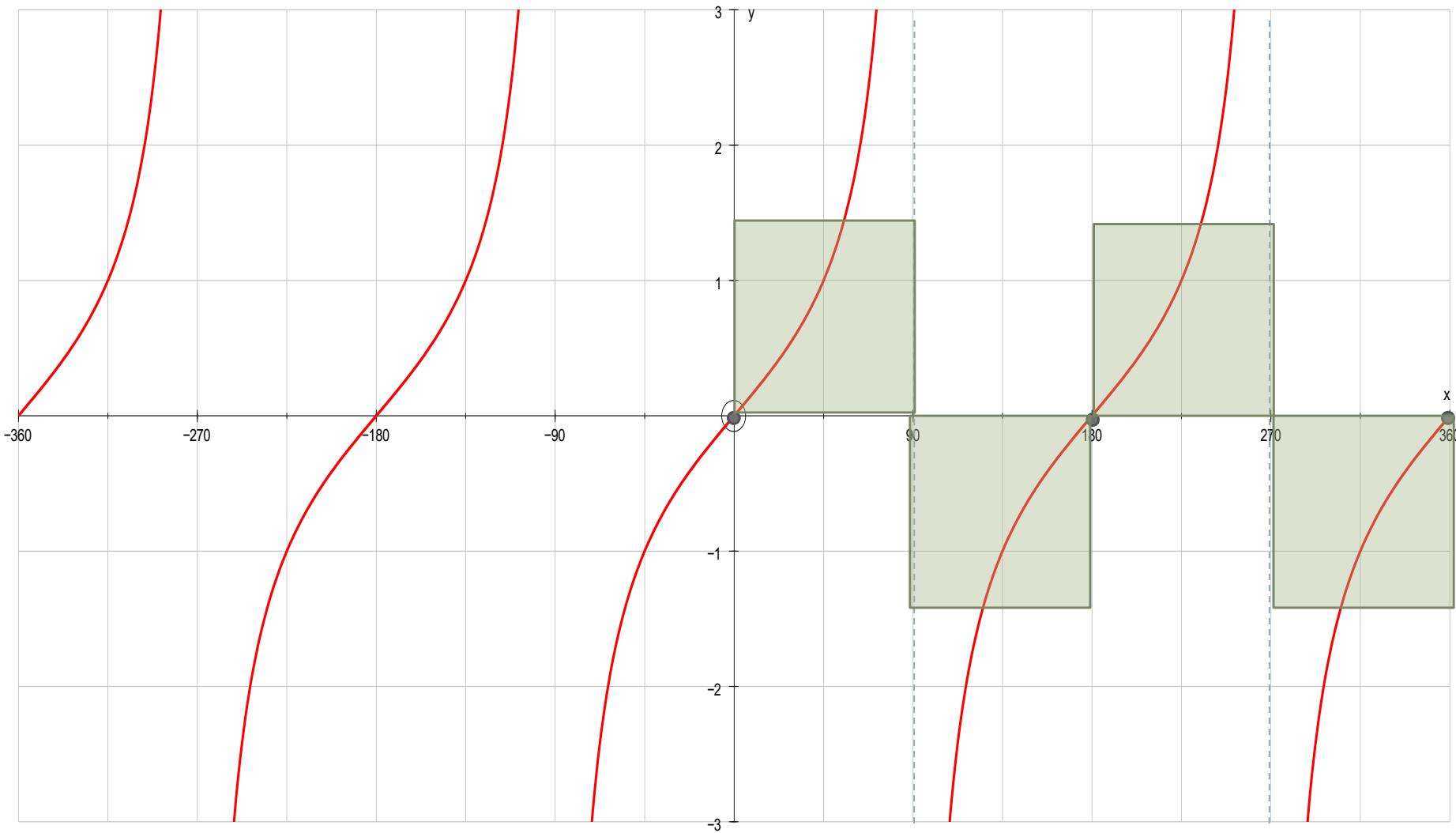
# Trigonometrie

Teken die funksie  $y = \cos x$

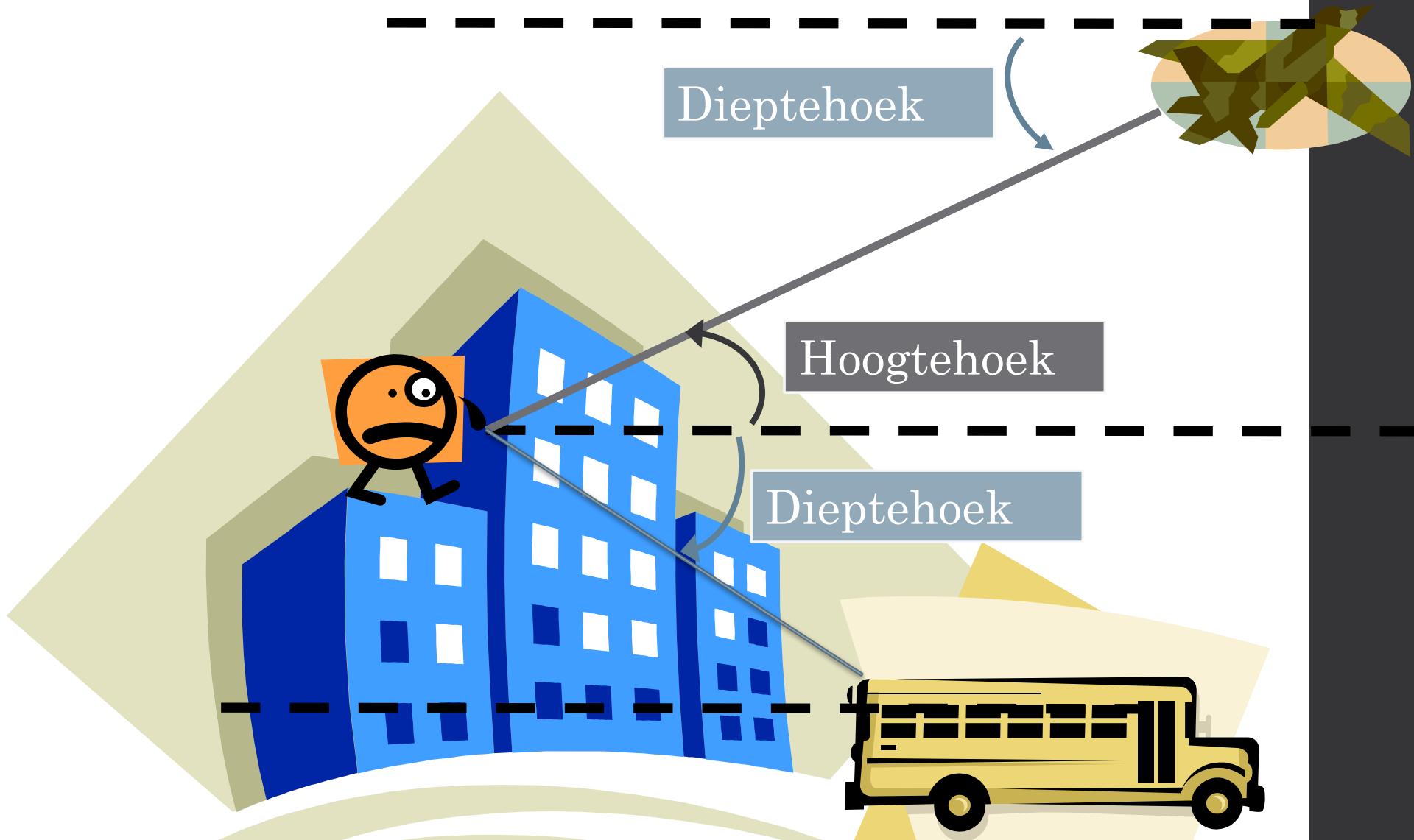


# Trigonometrie

Teken die funksie  $y = \tan x$



# Hoogte en dieptehoek



# TRIGONOMETRIE

Gr 11

1ste en 3de  
kwartaal

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$$\cos 15^\circ$$

$$= \frac{\pm \sqrt{1 - t^2}}{1}$$

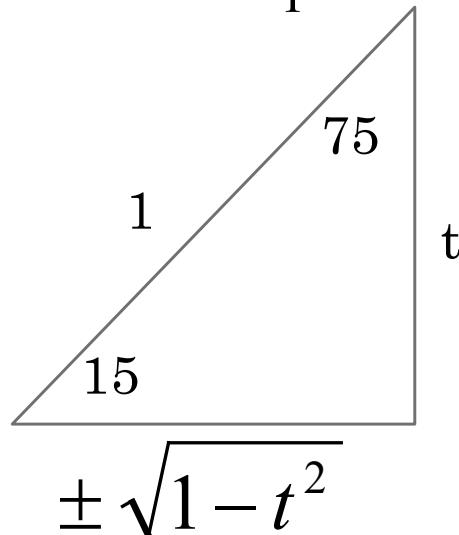
$$x^2 = 1^2 - t^2$$

$$= 1 - t^2$$

$$x = \pm \sqrt{1 - t^2}$$

$$\sin 15^\circ = t$$

$$\therefore \sin 15^\circ = \frac{t}{1}$$



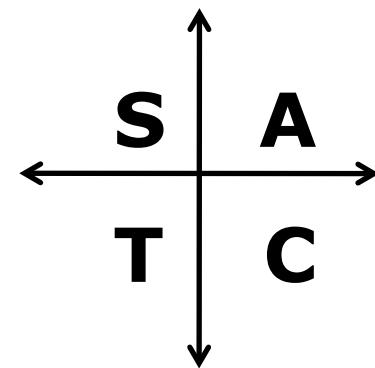
$$\pm \sqrt{1 - t^2}$$

Druk die volgende uit in terme van  $t$  as  $t = \sin 15$ :

$$\sin(-30^\circ)$$

Teken

= -



$$\sin(-30^\circ)$$

Funksie

$$= -\sin$$

$$\sin(-30^\circ)$$

Skerphoek

$$= -\sin 30^\circ$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\tan x = \frac{t}{s} \div \frac{a}{s}$$

$$\tan x = \frac{t}{s} \times \frac{s}{a}$$

$$\tan x = \frac{t}{a}$$

$$\sin x = \frac{t}{s}$$

$$\cos x = \frac{a}{s}$$

# Identiteite

$$\sin^2 x + \cos^2 x = 1$$

$$\left(\frac{t}{s}\right)^2 + \left(\frac{a}{s}\right)^2$$

$$= \frac{t^2 + a^2}{s^2}$$

$$= \frac{s^2}{s^2}$$

$$\sin x = \frac{t}{s}$$

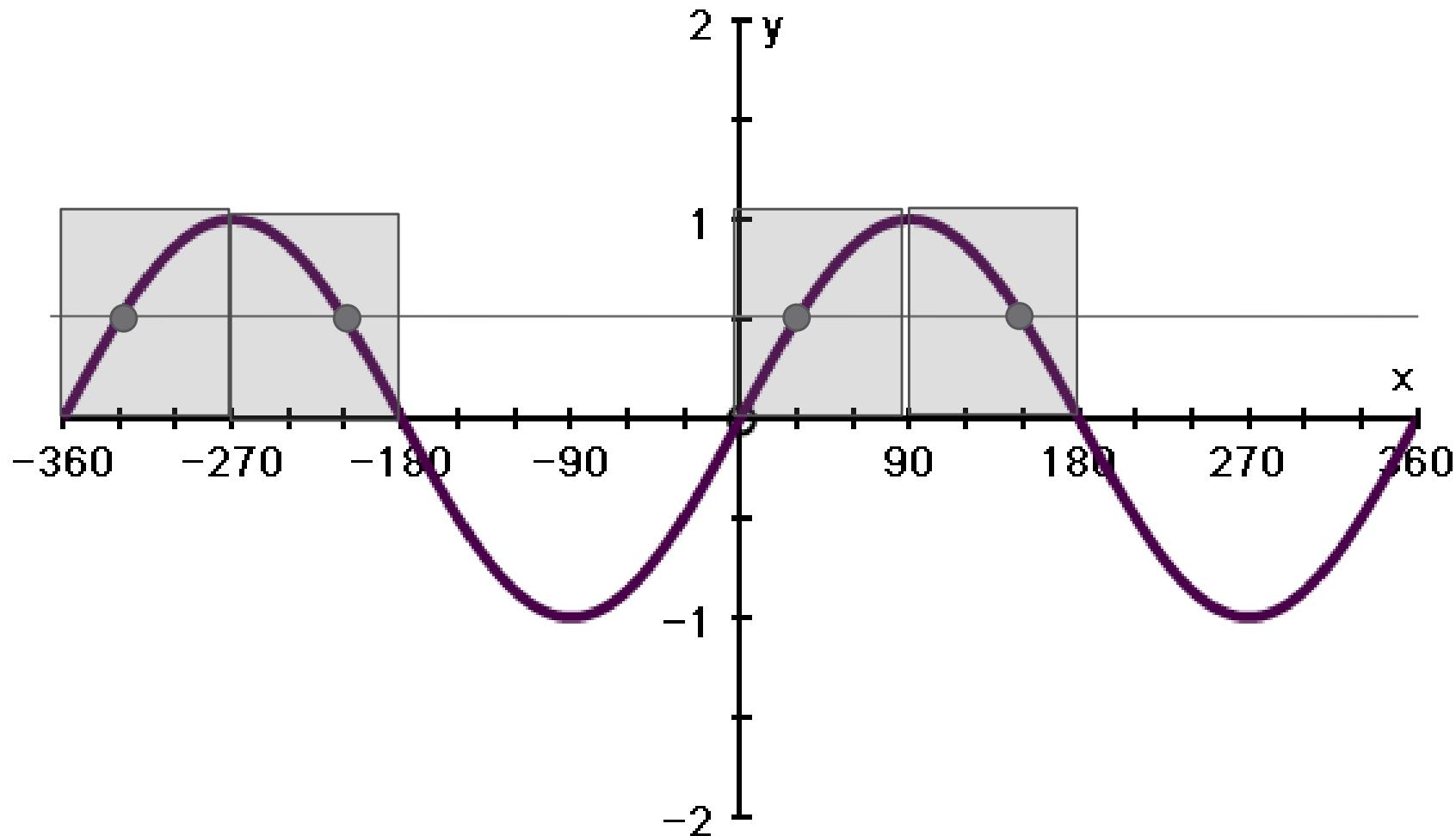
$$\cos x = \frac{a}{s}$$

Pythagora

**s**

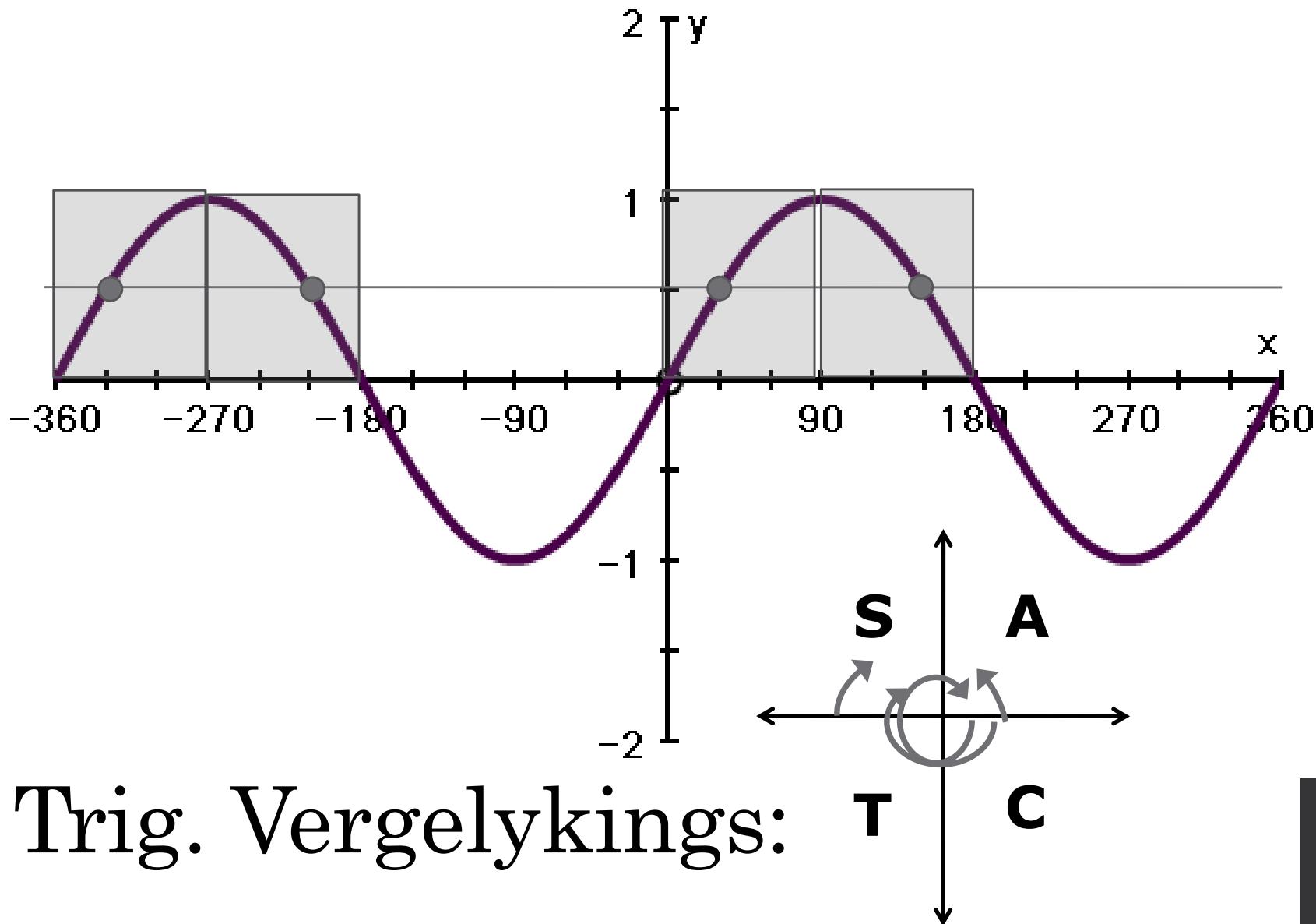
# Identiteite

Waar is  $\sin x = 0.5$  ???

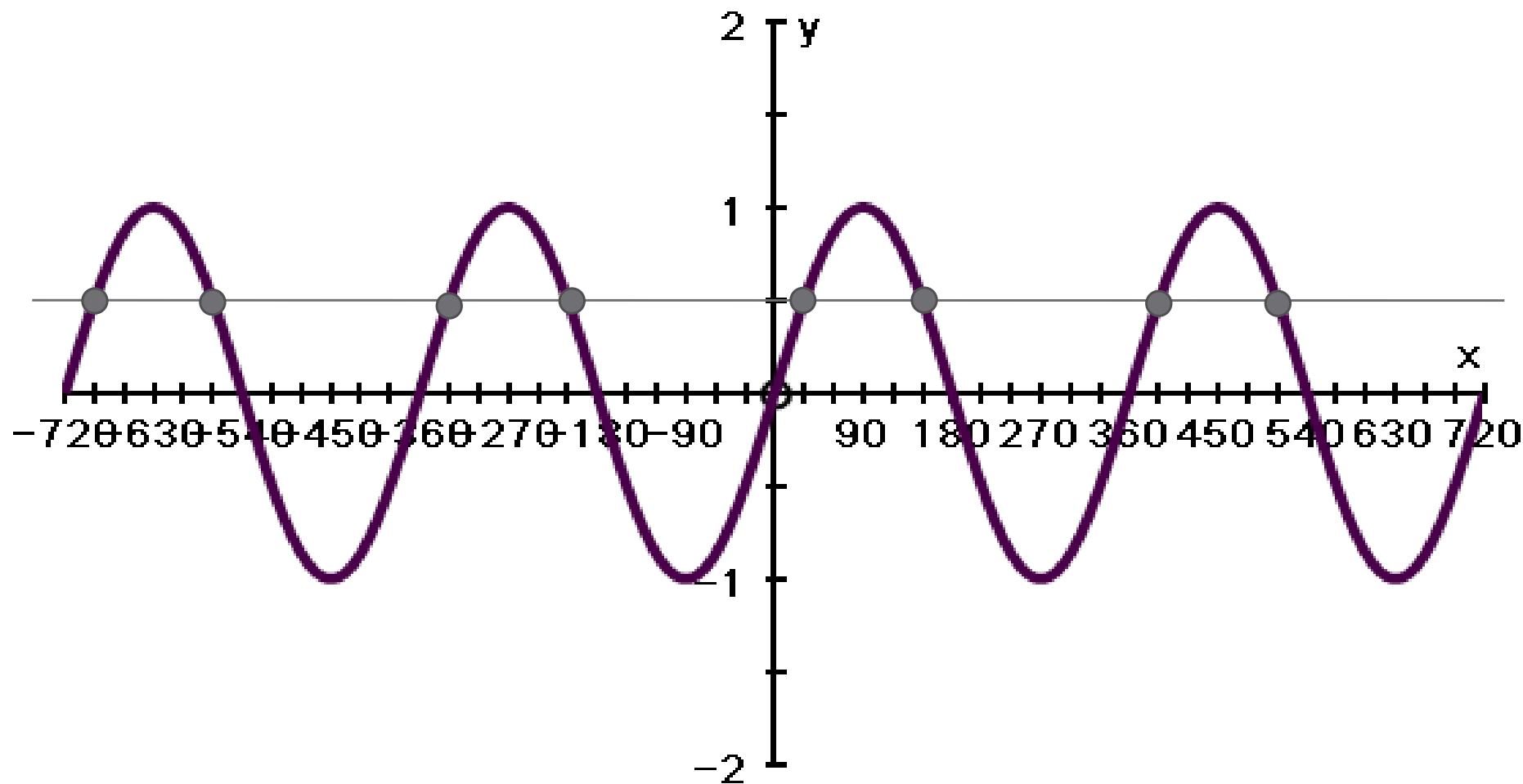


Trig. Vergelykings:

Waar is  $\sin x = 0.5$  ???



Waar is  $\sin x = 0.5$  ???



Trig. Vergelykings:

Bepaal die algemene oplossing vir  $x$  in die vergelyking

$$2 \cos 2x = -\sqrt{3}$$

waarde

$$\cos 2x = -\frac{\sqrt{3}}{2}$$

hoek

waarde

Kyk nou na teken:

Verwysingshoek =  $30^\circ$

III

II

$$2x = 180^\circ + 30^\circ + k \cdot 360^\circ$$

$$2x = 180^\circ - 30^\circ + k \cdot 360^\circ$$

$$2x = 210^\circ + k \cdot 360^\circ$$

$$2x = 150^\circ + k \cdot 360^\circ$$

$$x = 105^\circ + k \cdot 180^\circ$$

$$x = 75^\circ + k \cdot 180^\circ$$



Dieselde funksie

$$\sin(3x) = -\sin x$$

$\text{Sin}x/\text{Cos}x = \text{Tan}x$

$$3 \sin x = 4 \cos x$$

Ko-funksie

$$\cos 2\theta = \sin(\theta - 21^\circ)$$

$$\cos 2\theta = \cos[90 - (\theta - 21^\circ)]$$

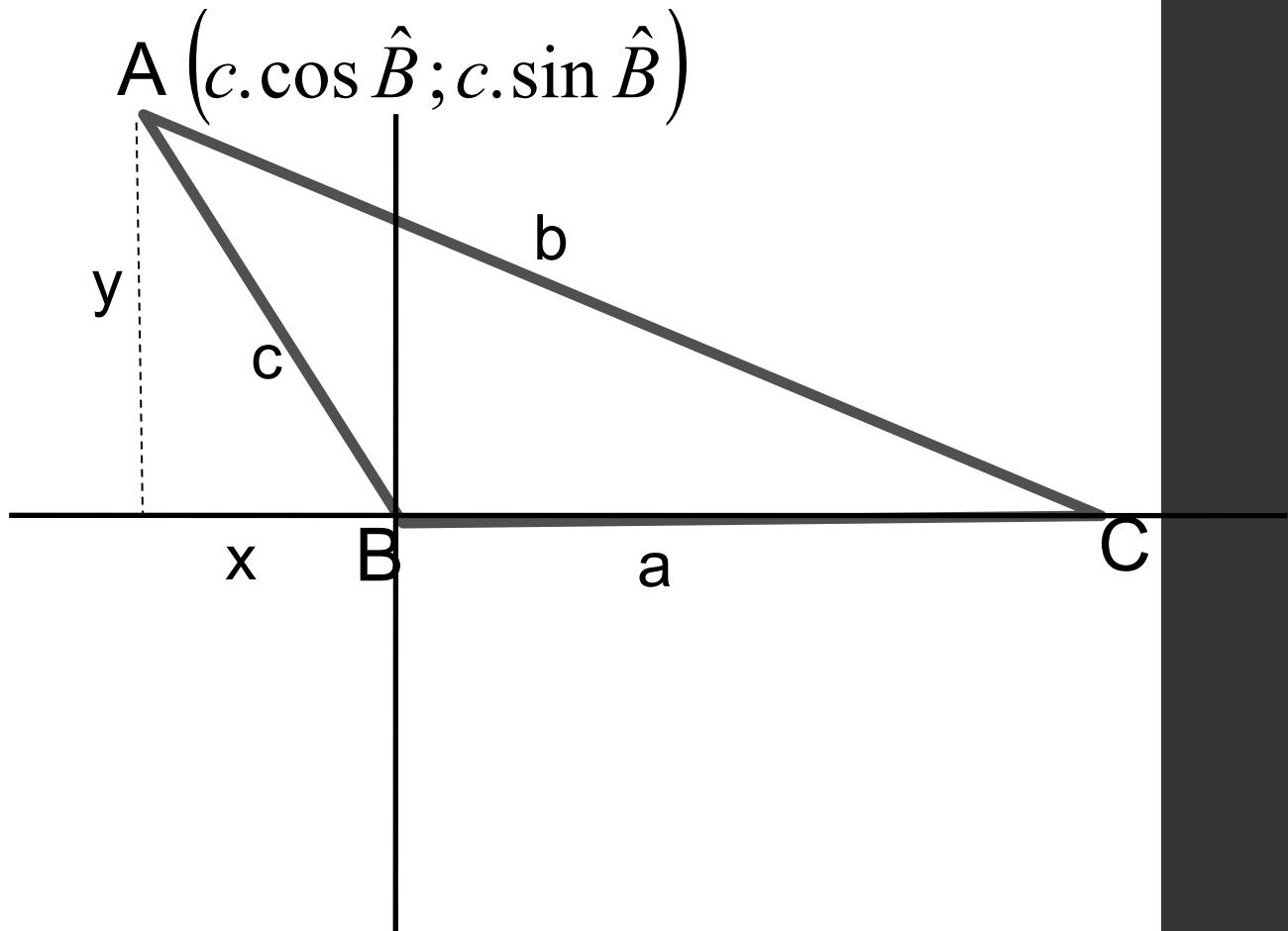
# Oppervlak reël

$$\cos \hat{B} = \frac{x}{c}$$

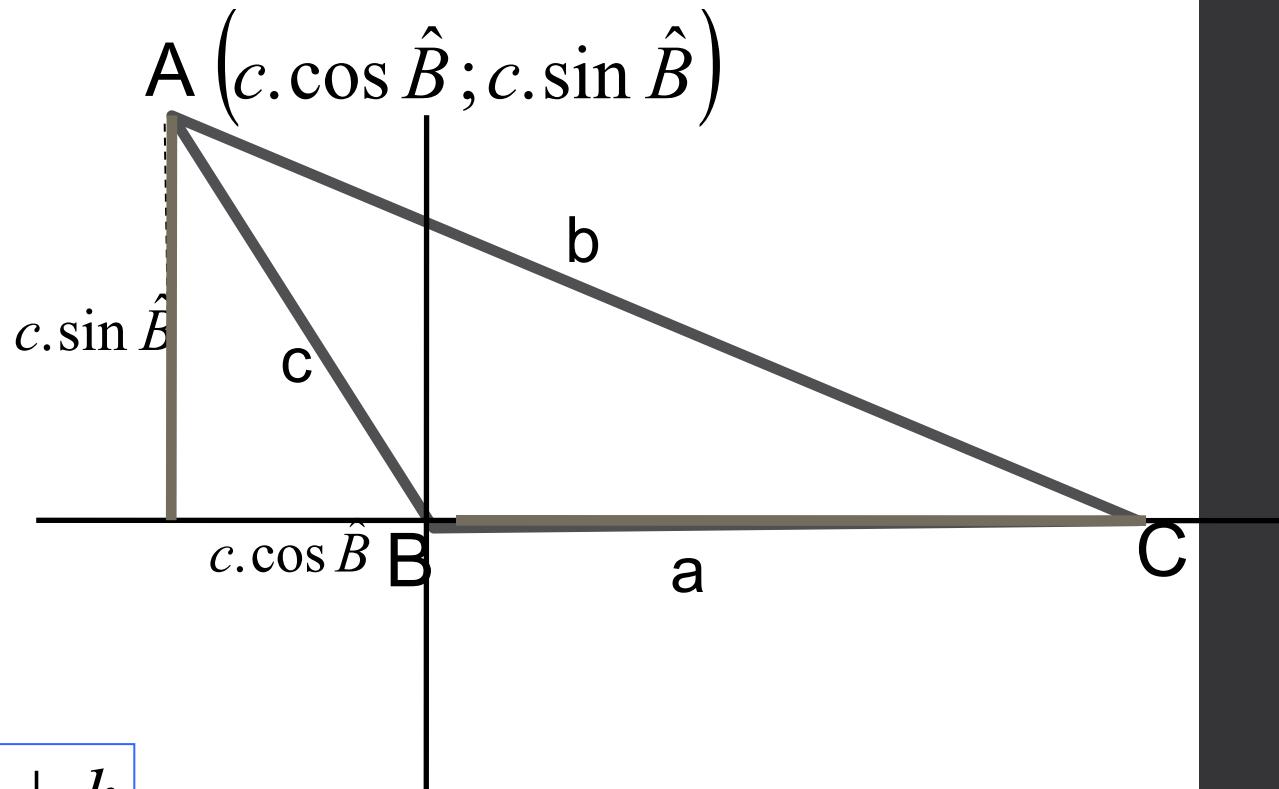
$$c \cdot \cos \hat{B} = x$$

$$\sin \hat{B} = \frac{y}{c}$$

$$c \cdot \sin \hat{B} = y$$



# Oppervlak reël



$$\text{Oppervlak} = \frac{1}{2} b \cdot h$$

$$\text{Oppervlak} = \frac{1}{2} a \cdot c \cdot \sin \hat{B}$$

# Oppervlak reël

$$Oppervlak = \frac{1}{2} a.c.\sin \hat{B}$$

$$Oppervlak = \frac{1}{2} b.c.\sin \hat{A}$$

$$Oppervlak = \frac{1}{2} a.b.\sin \hat{C}$$

# Sinus reël

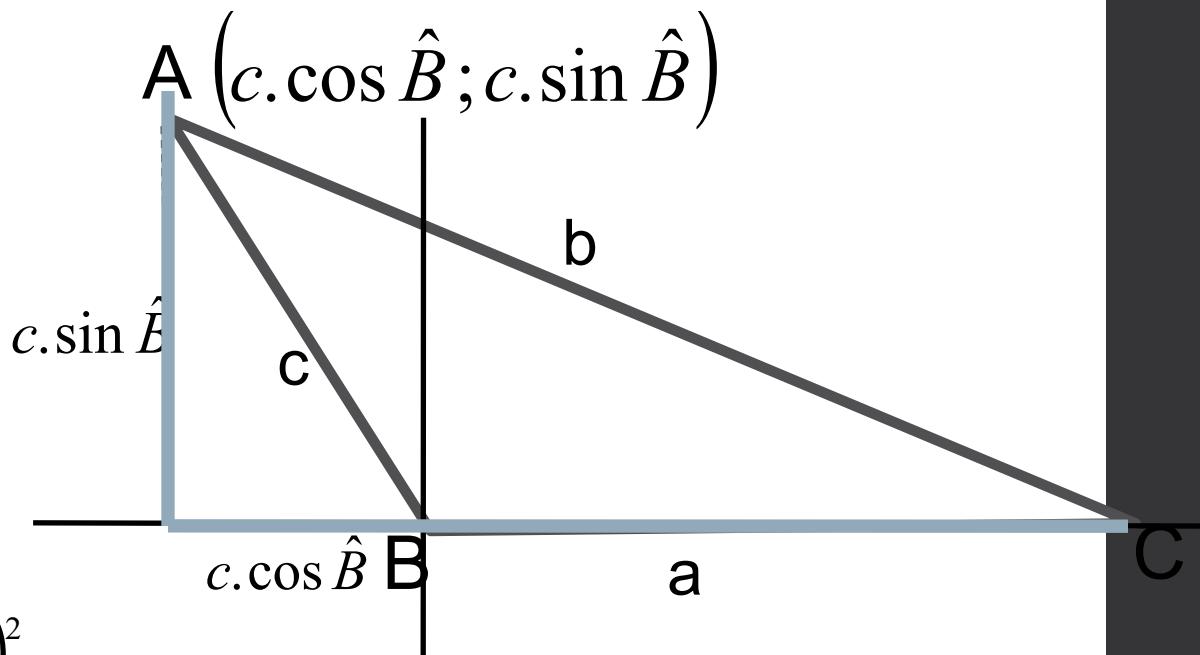
$$\div \frac{1}{2} abc :$$

$$Oppervlak = \frac{\frac{1}{2} a.c.\sin \hat{B}}{\frac{1}{2} abc} = \frac{\frac{1}{2} b.c.\sin \hat{A}}{\frac{1}{2} abc} = \frac{\frac{1}{2} a.b.\sin \hat{C}}{\frac{1}{2} abc}$$

$$\frac{\sin \hat{B}}{b} = \frac{\sin \hat{A}}{a} = \frac{\sin \hat{C}}{c}$$

$$\frac{b}{\sin \hat{B}} = \frac{a}{\sin \hat{A}} = \frac{c}{\sin \hat{C}}$$

# Kosinusreël



PYTHAGORAS:

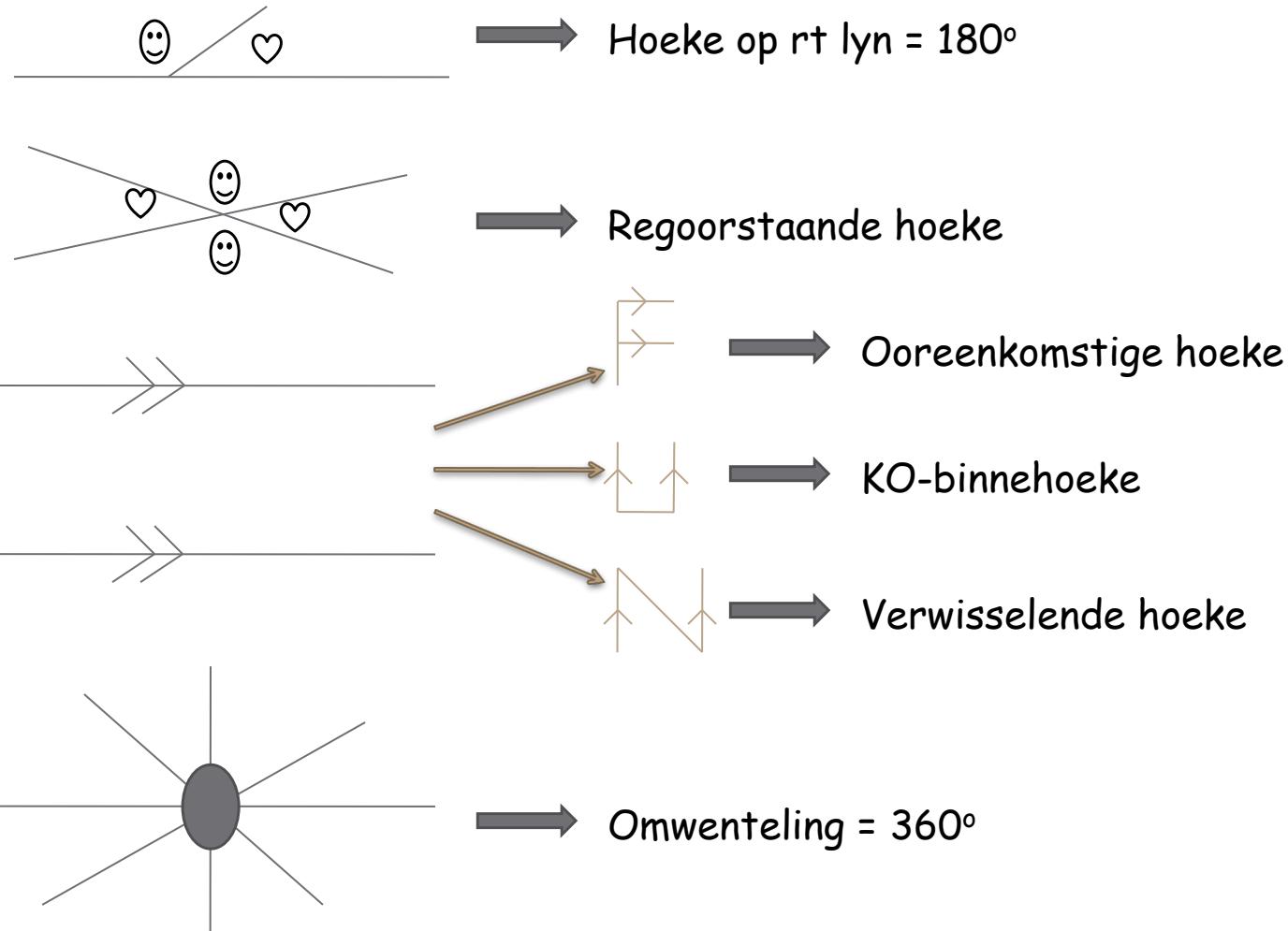
$$\begin{aligned} b^2 &= (a - c \cdot \cos \hat{B})^2 + (c \cdot \sin \hat{B})^2 \\ &= a^2 - 2ac \cdot \cos \hat{B} + c^2 \cdot \cos^2 \hat{B} + c^2 \cdot \sin^2 \hat{B} \\ &= a^2 - 2ac \cdot \cos \hat{B} + c^2 (\cos^2 \hat{B} + \sin^2 \hat{B}) \\ &= a^2 + c^2 - 2ac \cdot \cos \hat{B} \end{aligned}$$

# MEETKUNDE

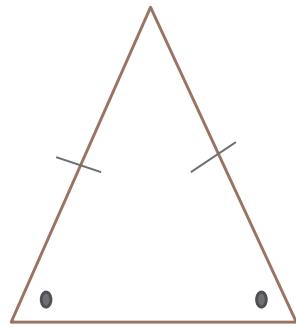
1ste & 3de  
kwartaal

Gr 10

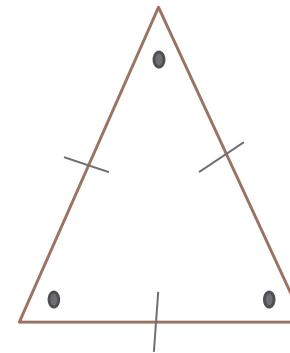
# Reguitlyn meetkunde



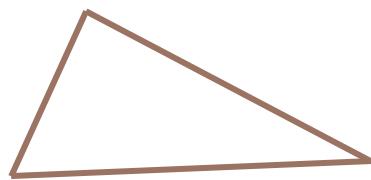
# Driehoeke



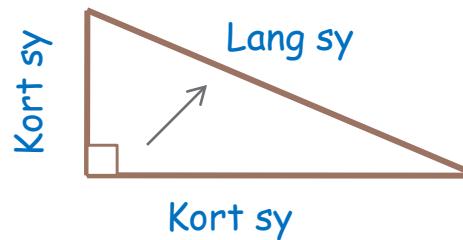
Gelykbenige  $\Delta$



Gelyksydige  $\Delta$



Ongelyksydige  $\Delta$

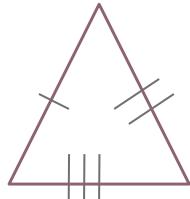


$$\text{Lang sy}^2 = \text{kort sy}^2 + \text{kort sy}^2$$

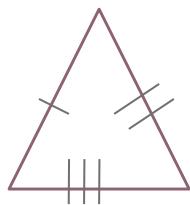
$$\text{Kort sy}^2 = \text{Lang sy}^2 - \text{kort sy}^2$$

PYTH

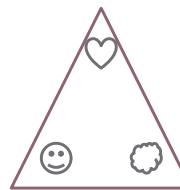
# Driehoede



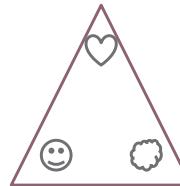
Kongruensie



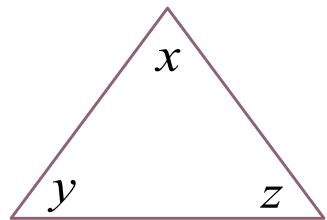
- SSS
- SHS
- HHS
- $90^\circ$ ; sk; s



Gelykvormigheid

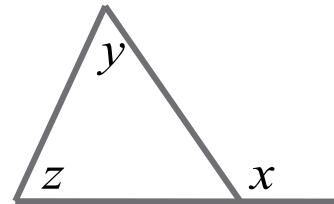


- HHH
- Sye in verhoudig



$$x + y + z = 180^\circ$$

Binnehoede van  $\Delta$



$$x = y + z$$



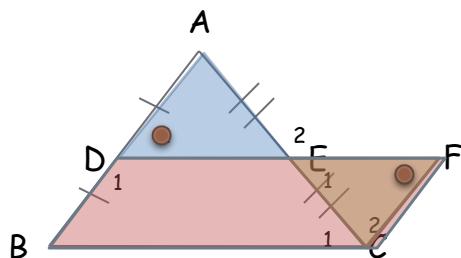
Buitehoek = som van t/o binnehoede

## 5 Maniere om te bewys 'n vierhoek is 'n parallelogram:

1. Teenoorstaande sye is ewewydig
2. Teenoorstaande sye is gelyk
3. Teenoorstaande hoeke is gelyk
4. Hoeklyne halveer mekaar
5. Een paar teenoorstaande sye is gelyk en ewewydig

## Voorbeeld 1:

Die lyn wat die middelpunt van twee sye van 'n driehoek verbind, is **ewequivalens aan die derde sy** en **gelyk aan die helfte van die lengte van die derde sy**.



Verleng  $DE$  se lengte tot  $F$ .

Verbind  $FC$ .

In  $\triangle EAD$  en  $\triangle ECF$  is:

$\hat{E}_1 = \hat{E}_2$       Regoorstaande hoeke

$AE = CE$       Gegee

$DE = EF$       Konstruksie

$\therefore \triangle EAD \equiv \triangle ECF$  SHS

$$E\hat{D}A = E\hat{F}C$$

$\therefore BD // FC$  (Verwisselende hoeke gelyk)

$$BD = DA \quad (\text{Gegee})$$

$$\therefore BD = FC \quad (\triangle EAD \cong \triangle ECF)$$

$DCFB$  is  $//m$       (1 paar sye = en  $//$ )

$$DF = BC$$

$$\text{d.w.s } DE = \frac{1}{2}BC$$

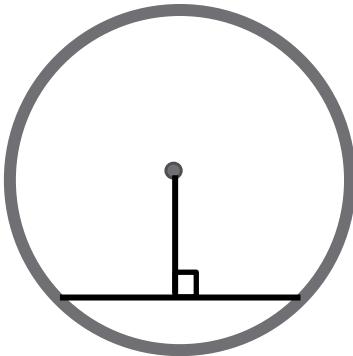
**d.w.s**  $DE // BC$

# MEETKUNDE

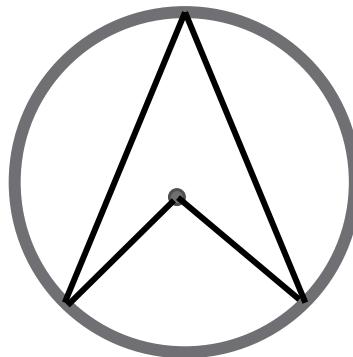


Gr 11

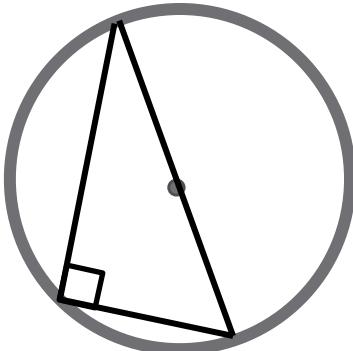
## Middelpunte



- 1) Loodlyn uit mdpt na koord
- 2) Middelloodlyn van 'n koord

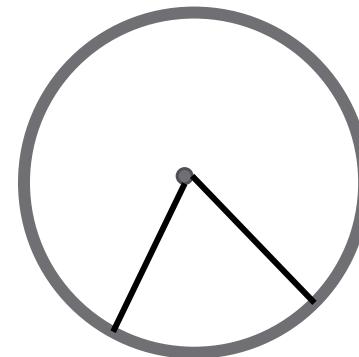


- 3) Midt hoek = 2x omtreks hoek



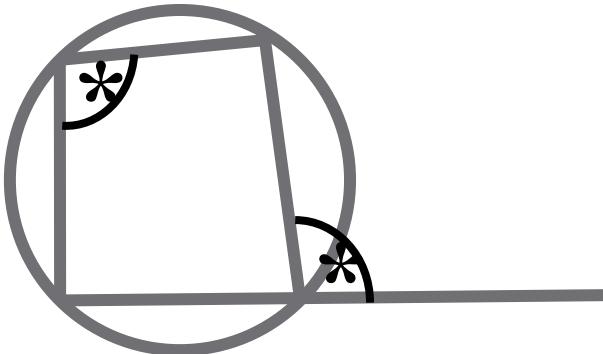
- 4) Middellyn onderspan  $90^0$  hoek

## Radiusse

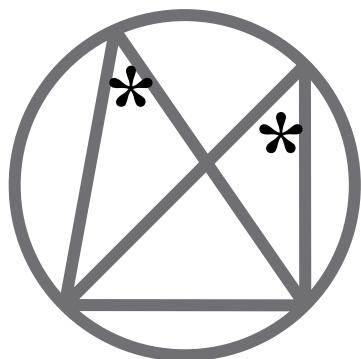


- 5) Radiusse

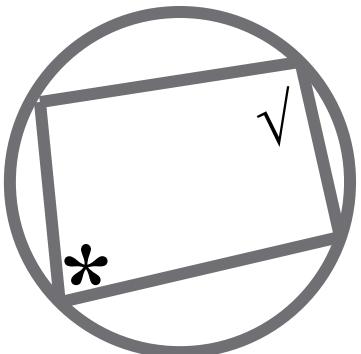
## Koordevierhoeke



1) Buitehoek = t/o binnehoek



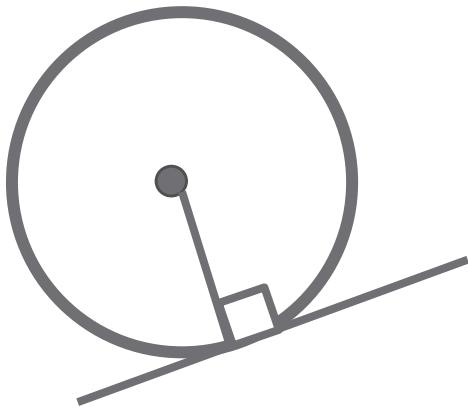
2) Omtreks hoeke in dieselfde segment is gelyk  
(omtrekshoeke onderspan deur dieselfde koord  
is gelyk)



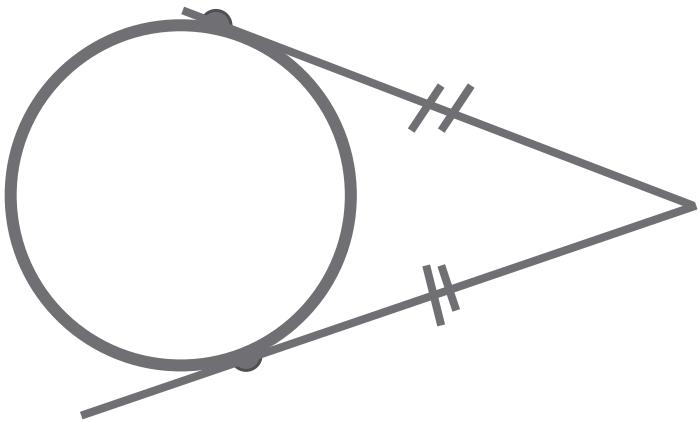
3) Teenoorstaande hoeke is supplementêr.

$$* + \sqrt{ } = 180^{\circ}$$

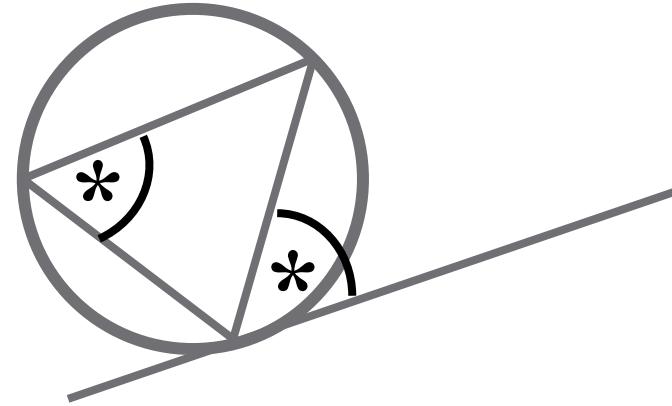
## Raaklyne



1) Radius loodreg op raaklyn

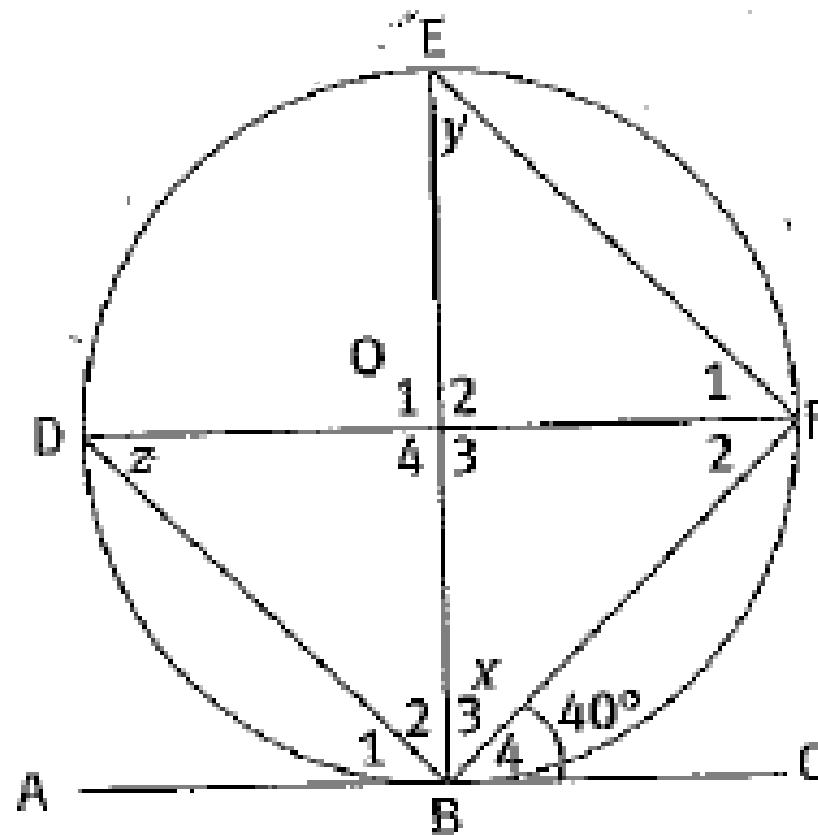


2) Raaklyne vanaf dieselfde punt is ewe lank



3) Hoek tussen raaklyn en koord is ewe groot.

O is die middelpunt van die sirkel en AC is 'n raaklyn. Bepaal die waardes van x, y en z



Baie dankie vir u tyd!